Calculus II
 Review 2

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As announced, the test will be on Tuesday, 9 March 1999 from 7:30pm to 9:30pm. The exam for sections 003–005 will be in Biological Sciences 107.

You may use a graphing calculator during the exam. You may not use notes, the text or any sort of integral table on the exam. You will be expected to know all of the integrals in the table on page 375. (Didn't you learn them last semester as differentiation rules?) On the exam, you should try to explain your work as carefully and completely as you can. Students giving unsupported answers may not receive full credit, even if they give the correct answers. Students who present their work neatly and carefully may receive more partial credit than students whose papers are hard to read. If I can't read it, I can't give you credit for it!

The test will cover Appendix F, the handout on trigonometric integrals and trigonometric substitution, and section 6.1–6.5 of Stewart. As you review for the exam, you should make sure that you can work all assigned problems from the text and the quiz. Below are several questions to help review for the exam.

1. Find the area between the curves $x = 2y^2 + 1$ and $x = y^2 + y + 1$.

2. On the test, you will be given the double-angle formulae

$$\cos^2\theta - \sin^2\theta = \cos 2\theta \qquad 2\sin\theta\cos\theta = \sin 2\theta.$$

   Use one of these identities and the Pythagorean identity to express $\sin^2\theta$ in terms of $\cos 2\theta$.

3. If $\cos u = x$ and $-\pi < u < 0$, express $\sin u$, $\sec u$, $\tan u$, $\cos 2u$ and $\sin 2u$ in terms of x.

4. Find

$$\int_0^1 \sqrt{4 - x^2}\, dx.$$

   Use a geometric argument to check your answer.

5. Find a function F so that

$$F'(x) = \frac{1}{4 + x^2}.$$

6. Evaluate

   (a) $\displaystyle\int_0^{\pi/4} \sin^3 x\, dx$

   (b) $\displaystyle\int \sin^2 x\, dx$

(c) $\int x^2\sqrt{9-x^2}\,dx$

(d) $\int \frac{x}{4+x^2}\,dx$

(e) $\int \frac{x^2}{4+x^2}\,dx$

(f) $\int \frac{1}{x(x^2+1)}\,dx$

(g) $\int \tan x\,dx$

(h) $\int \cos^{-2} x\,dx$

(i) $\int \frac{1}{x^2-4}\,dx$

7. Find the partial fractions decomposition for the function

$$f(x) = \frac{x^2-1}{(x+2)(x+1)(x^2+2)}.$$

8. Give the definition of a rational function. (Are there irrational functions?)

9. Find the volume of a right-circular cone of radius ρ and height h.

10. In each of these the problems below, you should indicate how you are slicing up the region, find the volume or area of each slice, sum and take a limit to show how an integral is obtained.

    (a) Find the area between the curves $y = x^2 - 2x + 2$ and $y = 2$.

    (b) Find the volume obtained by rotating the area in part a) about the y-axis.

    (c) Find the volume obtained by rotating the area in part a) about the x-axis.

11. Find the arclength along the curve

$$y = \sqrt{1-x^2}, \qquad 0 \le x \le 1/2.$$

You should be able to check your answer geometrically.

12. Find the arclength along the curve $x(t) = t^2$, $y(t) = t^3$, $2 \le t \le 4$.

13. Find a number c so that

$$\int_0^{\pi} c + \sin x\,dx = 0.$$

14. Find the volume of the spherical cap obtained by slicing the top 3 inches off of a soccer ball of radius 5 inches.

15. Problems involving work such as §6.5, #13–16 and hydrostatic pressure such as §6.5, #19–23. (You've already done most of these, right?) In each case, you should be able explain how to divide the problem into bits, determine the work or force corresponding to each bit, sum and take limits to arrive at an integral. Can you give the correct units?