Calculus II — Russell Brown
Exam 1 — 9 February 1998

Answer all of the following questions on the paper that is handed out. Please be sure to put your name and section number on each page. Number your pages. Additional sheets are available if necessary. No books or notes may be used. When answering these questions, please be sure to 1) check answers when possible, 2) clearly indicate your answer and the reasoning used to arrive at that answer *(unsupported answers may receive NO credit)*.

If you use your calculator to solve an equation or produce a graph, please indicate this on your test paper. Otherwise the answer may be assumed to be "unsupported".

Name ______________________

Section __________

Question	Score	Total
1		10
2		15
3		10
4		35
5		15
6		15
Total		100

1. A soccer ball is rolling down a steep hill. At time $t = 0$ seconds, its velocity is 1 m/s. The table below gives its acceleration at several times from $t = 0$ to $t = 5$ seconds. Use this information to estimate the velocity at $t = 5$ seconds. (Several different answers are possible. Be sure and explain how you arrived at your answer.)

time (seconds)	acceleration (meter/second^2)
0	5
1	4
1.5	4
3	3
3.5	3
5	2.5

2. (a) State the extreme value theorem.

   (b) State the evaluation theorem and the fundamental theorem of calculus, part 1.

3. Use a geometric argument to compute the integral

$$\int_{-1}^{2} 1 - |x| \, dx.$$

   Hint: Draw a graph.

4. Use calculus to evaluate the following integrals:

   (a) $\displaystyle\int_2^3 (x-2)^5 \, dx$

   (b) $\displaystyle\int_0^\infty \frac{dx}{1+x^2}$

   (c) $\displaystyle\int_0^1 x\sqrt{2-x^2} \, dx$

   (d) $\displaystyle\int xe^{2x} \, dx$

   (e) $\displaystyle\int e^x \cos x \, dx$

5. The error in Simpson's rule with n intervals satisfies

$$|S_n - \int_a^b f(x)\,dx| \le \frac{K_4(b-a)^3}{180n^4}.$$

Here, K_4 is an upper bound for the fourth derivative of f on the interval $[a,b]$, that is we have $|f^{(4)}(x)| \le K_4$, for $a \le x \le b$.

(a) Use Simpson's rule with $n = 4$ to find an estimate for

$$\int_1^4 \frac{dx}{x}.$$

(b) Let $f(x) = 1/x$ and find the fourth derivative $f^{(4)}(x)$.

(c) Use the formula above to give an estimate of the error between your answer to part a) and the exact value of the integral.

(d) How many terms do we need to find the integral in part a) accurate to within 10^{-8}?

6. (a) Is

$$\int_0^{\pi/2} \tan x\,dx$$

a convergent integral? If so, give its value.

(b) Is

$$\int_0^2 \frac{1}{\sqrt{4-x^2}}\,dx$$

a convergent integral? If so, give its value.