MA114:012–014 Russell Brown
Exam 2 10 March 1998

Answer all of the following questions. Use the backs of the question papers for scratch paper. Additional sheets are available if necessary. No books or notes may be used. When answering these questions, please be sure to 1) check answers when possible, 2) clearly indicate your answer and the reasoning used to arrive at that answer **(unsupported answers may receive NO credit)**, and 3) label all variables and equations.
If you use your calculator to solve an equation or produce a graph, please indicate this on your test paper. Otherwise the answer will be assumed to be "unsupported".

Name _____

Section _____

Question	Score	Total
1		15
2		15
3		15
4		15
5		10
6		15
7		15
Total		100

1. (a) Use Simpson's rule with $n = 6$ to estimate the integral:

$$\int_1^2 \sin(2x)\,dx.$$

(b) Recall that the error in Simpson's rule is estimated by

$$E_S = \left| \int_a^b f(x)\,dx - S_n \right| \leq \frac{(b-a)^5 K_4}{180 n^4}$$

where K_4 is a number which satisfies $f^(4)(x) \leq K_4$ for all x, $a \leq x \leq b$ and n is the number of subdivisions. Find a value of n so that $E_S \leq 0.0001$ for the integral in part a).

2. Compute the following improper integrals, if they are convergent. If an integral is divergent, explain why.

   (a) $\displaystyle\int_1^\infty \frac{dx}{x^2}$

   (b) $\displaystyle\int_{-\infty}^\infty \frac{dx}{1+x^2}$

   (c) $\displaystyle\int_{-\infty}^\infty \frac{x}{1+x^2}\,dx$

3. Let R be the region in the first quadrant bounded by $y = x^3$ and $y = 2x - x^2$. Find the area of R.

4. Let P be a pyramid with a square base which is 4 meters by 4 meters. Suppose the height of P is 6 meters. Find the volume of the solid obtained by slicing P in the plane which is parallel to the base and 3 meters above the base. As part of your answer, you should show how to express the volume as an integral.

5. Find a number c so that

$$\int_{-1}^{2} (x^2 - 2x - c)\, dx = 0.$$

6. Suppose we have a tank which is a right circular cone with depth 5 meters and radius 2 meters. The tank is full of water of density $\rho = 1000\ \mathrm{kg/m^3}$. Find the work done in pumping all the water out of the tank through an outlet at at the top of the tank. As part of your answer, you should show how to express the total work as an integral. You may need to use that the acceleration of gravity, $g = 9.8\ \mathrm{m/s^2}$.

7. Consider the differential equation

$$y' = y^3 - y.$$

(a) Find all the equilibrium solutions, $y(x) = c$.

(b) Sketch a direction field for this equation.

(c) Use your answer to part b) to sketch the solution of $y' = y^3 - y$ and $y(0) = -4$.