MA114:012–014 Russell Brown
Exam 3 14 April 1998

Answer all of the following questions. Use the backs of the question papers for scratch paper. Additional sheets are available if necessary. No books or notes may be used. When answering these questions, please be sure to 1) check answers when possible, 2) clearly indicate your answer and the reasoning used to arrive at that answer **(unsupported answers may receive NO credit)**, and 3) label all variables and equations.
Please indicate where and how you use your calculator to produce an answer. In general, you are to use the methods introduced in this course to solve problems.

Name ______________________

Section ____________

Question	Score	Total
1		15
2		15
3		10
4		15
5		15
6		15
7		15
Total		100

1. (a) Find a function $y(x)$ which satisfies

$$y' = 6 - 3y, \qquad y(0) = 0.$$

(b) For $y(x)$, the function you found in part a), compute

$$\lim_{x \to \infty} y(x).$$

2. Consider the family of curves

$$2x^2 + 3y^2 = c.$$

(a) Use implicit differentiation to find a differential equation that these curves satisfy.

(b) Write a differential equation for the trajectories which are orthogonal to our family of curves.

(c) Solve the differential equation you found in part (b) to find the orthogonal trajectories.

3. For each of the following sequences $\{a_n\}_{n=1}^{\infty}$, determine if

$$\lim_{n\to\infty} a_n$$

exists. If the limit exists, give the value of the limit.

(a) $a_n = \cos(n^2 + 1)/n$

(b) $a_n = e^n$

(c) $a_n = ne^{-n}$

4. (a) State the integral test for convergence of a series.

(b) Use the integral test to find an integer N so that

$$\left| \sum_{n=1}^{\infty} \frac{1}{n^{3/2}} - \sum_{n=1}^{N} \frac{1}{n^{3/2}} \right| \leq 10^{-4}.$$

5. (a) Find the sum of the series

$$\sum_{n=4}^{\infty} \frac{2}{3^n}.$$

(b) Find the sum

$$\sum_{n=4}^{221} 1.1^n.$$

(c) Express the repeating decimal

$$11.82828282\ldots$$

as a fraction

6. Test each of the following series for convergence. Your answer must indicate which test you use and how the test is applied.

   (a) $\sum_{n=1}^{\infty}(\cos(\pi(n+1)) - \cos(\pi n))$

   (b) $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$

   (c) $\sum_{n=1}^{\infty} \frac{n}{3.1^n}$

7. For the following power series, give the radius of convergence and the interval of convergence. Justify your answer.

(a) $\sum_{n=1}^{\infty} \frac{x^n}{n!}$

(b) $\sum_{n=1}^{\infty} \frac{(x-3)^n}{n}$