Calculus II — Russell Brown
Exam 3 — 13 April 1999

Answer all of the following questions. Use the answer sheets provided. Additional sheets are available if necessary. No books or notes may be used. You may use a calculator. When answering these questions, please be sure to 1) check answers when possible, 2) clearly indicate your answer and the reasoning used to arrive at that answer **(unsupported answers may receive NO credit)**, and 3) label all variables and equations.
Note that the point total on this exam is greater than 100 points, however no student will be given a grade of greater than 100.

Name ________________________

Section ____________

Question	Score	Total
1		15
2		10
3		20
4		20
5		15
6		20
7		10
Total		110
min(total,100)		100

1. Find the limits of the following sequences.

   (a) $\lim_{n\to\infty} \frac{n^2+1}{2n^2+n}$

   (b) $\lim_{n\to\infty} \frac{1}{1+2^{-n}}$

   (c) $\lim_{n\to\infty} \frac{1}{1+2^{n}}$

2. Suppose that a sequence $a_1, a_2, \ldots$ is defined by

$$a_{n+1} = 4 - \frac{1}{a_n} \quad \text{and} \quad a_1 = 1.$$

   It can be shown that this sequence is bounded and increasing. Assuming that the sequence is bounded and increasing, explain why the sequence has a limit and compute the exact value of the limit

$$\lim_{n\to\infty} a_n.$$

   (You may check your answer by finding an approximate value for the limit using your calculator.)

3. Determine if each of the following series converges and, for the convergent series, compute the sum of the series.

   (a) $\sum_{n=0}^{\infty} \frac{2^n}{n!}$

   (b) $\sum_{n=4}^{\infty} 2^{-n}$

   (c) $\sum_{n=0}^{\infty} 3^n$

   (d) $\sum_{n=1}^{\infty} \left(\frac{1}{n} - \frac{1}{n+2}\right)$

4. (a) Express

$$\int e^{-x^2}\,dx$$

   as a power series.

(b) Give the interval of convergence of the series you found in part a).

(c) Use the series in a) to express the definite integral $\int_0^{1/2} e^{-x^2}\,dx$ as a series. Determine how many terms are needed to approximate the integral to within $1/500$ and evaluate a partial sum of the series, s_N, which satisfies

$$\left| \int_0^{1/2} e^{-x^2}\,dx - s_N \right| < 1/500.$$

5. (a) State the integral test for convergence of a series.

   (b) What N is needed so that

$$\left| \sum_{n=1}^{\infty} \frac{1}{n^5} - \sum_{n=1}^{N} \frac{1}{n^5} \right| < 10^{-5}?$$

   Do not evaluate the partial sum.

6. (a) Let

$$f(x) = \sqrt[3]{1+x}$$

   and compute f', f'' and $f^{(3)}$.

   (b) Guess a formula for $f^{(n)}(x)$.

   (c) Prove that your guess is correct using the principle of mathematical induction.

   (d) Using your answer to b), write the MacLaurin series for $f(x)$. (Of course, this is a special case of the binomial series.)

7. For which values of x does the series

$$\sum_{n=1}^{\infty} \frac{x^n}{n}$$

converge.