1. Suppose that the supply and demand curves for 162 textbooks are

$$q = 10p \quad \text{(supply)}$$

$$q = 3000 - 50p \quad \text{(demand)}$$

a) Determine the equilibrium price and the quantity sold.

b) If the price is set at \$40, determine if there will be a shortage or a surplus. Give the quantity of shortage or surplus.

2. Suppose that the manufacture of one 162 textbook requires 289 sheets of paper, 2 oz. of glue and 1.1 ft^2 of of cover board. The manufacture of one 123 text requires 148 sheets of paper, 0.8 oz. of glue and 1.5 ft^2 of cover board. The manufacture of one 113 text requires 312 sheets of paper, 3.1 oz. of glue and 1.9 ft^2 of cover board. A publishing company has 50,000 sheets of paper, 300 oz. of glue and 150 ft^2 of cover board. Write down a system of equation whose solution is the number of 162, 123 and 113 textbooks that can be made. Do not solve the system.

3. Let $A = \begin{pmatrix} 1 & 2 \\ 2 & -1 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 2 & 3 \\ -3 & 2 & 1 \end{pmatrix}$, and $C = \begin{pmatrix} 1 & 2 \\ 3 & 4 \\ 1 & 2 \end{pmatrix}$.

Compute each of the following.

a) A^{-1}

b) $A \cdot B$

c) $A \cdot C$

d) $C + B$

4. The system

$$\begin{array}{rrrl} x & & -z & = 1 \\ x & +y & +z & = 2 \\ 3x & +y & -z & = 4 \end{array}$$

has infinitely many solutions.

a) Suppose that (x, y, z) is a solution. If z is arbitrary express x and y in terms of z.

b) Suppose that $z = 1$. Find the corresponding solution.

5. A student has math and English to study. Her total study time is 8 hours. In order to complete her English paper, she needs a minimum of 2 hours. Finally, she wishes to spend at least twice as long on math as on English.

a) Write a system of linear inequalities which summarize the above information.

b) Graph the system.

c) Label all corner points of the feasible set and give the coordinates of each corner point.

6. Plot the following supply and demand data on one graph and join the points with a smooth curve. Use your *graph* to answer the following questions.

Price in \$	Quantity supplied	Quantity demanded
\$10	3,000	8,000
\$12	5,000	7,000
\$15	6,000	5,500
\$20	7,000	3,000

(a) Do you think that either the supply curve or the demand curve is given by a linear function?

(b) Estimate the equilibrium price from your graph.

(c) For which prices is there surplus?

7. Find the solution using Gauss-Jordan elimination.

$$\begin{array}{rrrcr} x & & -z & = & -1 \\ x & +2y & +2z & = & 6 \\ x & -4y & +z & = & 1. \end{array}$$

8. (a) Write down a system that has no solutions.

(b) Consider the system

$$\begin{aligned} w + x + y + 2z &= 5 \\ w + 2x - y + 3z &= 5 \end{aligned}$$

i. Give all solutions to this system.

ii. Find two solutions to this system.

9. Compute the following matrix operations, if possible. If an operation is impossssible, explain why.

$$\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}\begin{pmatrix} 1 \\ 3 \end{pmatrix}, \quad \begin{pmatrix} 1 \\ 3 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}, \quad \begin{pmatrix} 1 \\ 3 \end{pmatrix} + \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$$

10. Mailing a letter costs x cents for the first ounce and y cents for each additional ounce. Suppose that a four ounce letter costs \$1.19 and a six ounce letter costs \$1.77.

(a) Write a linear system whose solution is x and y.

(b) Solve the system.

11. Find the solution by your favorite method.

$$\begin{array}{cccccc} x & +y & +z & = & 6 \\ x & -y & +z & = & 2 \\ x & +2y & -z & = & 2. \end{array}$$

12. Find the solution by your favorite method. Use the back if necessary.

$$\begin{array}{cccccc} x & & -z & = & -1 \\ x & +2y & +2z & = & 6 \\ x & -4y & +z & = & 1 \end{array}$$

13. Let A $= \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{pmatrix}$. Which of the following matrices is the inverse of A?

$$B = \begin{pmatrix} 1 & -1 & 0 \\ -1 & 1 & 1 \\ 1 & 0 & -1 \end{pmatrix} \quad \text{or} \quad C = \begin{pmatrix} 1 & -1 & 0 \\ -1 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix}$$

14. Plot the following supply and demand data on one graph and join the points with a smooth curve. Use your *graph* to answer the following questions.

Price in \$	Quantity supplied	Quantity demanded
\$10	3,000	8,000
\$12	5,000	7,000
\$15	6,000	5,500
\$20	7,000	3,000

(a) Do you think that either the supply curve or the demand curve is given by a linear function?

(b) Estimate the equilibrium price from your graph.

(c) For which prices is there surplus?

15. For each of the following matrix operations, either give the result or explain why it is impossible.

$$\begin{pmatrix} 1 & 2 \\ 4 & -1 \end{pmatrix}\begin{pmatrix} 2 \\ 3 \end{pmatrix}, \quad \begin{pmatrix} 1 & 2 \\ 4 & -1 \end{pmatrix}+\begin{pmatrix} 2 \\ 3 \end{pmatrix}, \quad \begin{pmatrix} 2 \\ -1 \end{pmatrix}, \begin{pmatrix} 2 & 3 & rr \end{pmatrix}.$$

16. Compute the following matrix operations, if possible. If an operation is imposssible, explain why.

$$\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}\begin{pmatrix} 1 \\ 3 \end{pmatrix}, \quad \begin{pmatrix} 1 \\ 3 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}, \quad \begin{pmatrix} 1 \\ 3 \end{pmatrix}+\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$$

1. Find the inverse of the matrix

$$\begin{pmatrix} 1 & 1 & 0 \\ 1 & 2 & 0 \\ 1 & 1 & 1 \end{pmatrix}$$

2. (a) Find two numbers whose sum is 22 and whose difference is 9. Use a 2×2 linear system to find your solution.

   (b) Write down a system of equations which has no solutions.

3. At the Smith family's favorite fast food restaurant, a hamburger contains 10 grams of fat, 70 grams of carbohydrates and 20 grams of protein. French fries contain 40 grams of fat, 50 grams of carbohydrates and 5 grams of protein. Milk shakes contain 60 grams of fat, 5 grams of carbohydrates and 15 grams of protein. Suppose that the Smiths have 200 grams of fat, 600 grams of carbohydrates and 150 grams of protein at lunch. Write down a system of equations whose solutions are the number french fries, milk shakes and hamburgers that the family had for dinner. Be sure to indicate what each variable in your system stands for. DO NOT solve the resulting system of equations.

4. Solve the system

$$\begin{aligned} 2x + y + z &= 3 \\ x + y + 2z &= 5 \\ x + y + z &= 2. \end{aligned}$$

5. Find two solutions to the system

$$\begin{aligned} x + y + z &= 7 \\ w + 2x + y \quad &= 7. \end{aligned}$$

6. (a) Find the inverse of the matrix

$$A = \begin{pmatrix} 1 & 2 & 0 \\ 1 & 1 & -1 \\ 1 & 2 & -1 \end{pmatrix}.$$

   (b) Use the inverse you found in part a) to solve the system

$$\begin{aligned} x + 2y \quad &= 3 \\ x + y - z &= 1 \\ x + 2y - z &= -1 \end{aligned}$$

7. Suppose that each ounce of food A has 2 grams of fat, 3 grams of carbohydrate and 2 units of Vitamin Z. Each ounce of food B has 3 grams of fat, 1 gram of carbohydrates and 3 units of vitamin Z. Suppose that one needs to consume at least 6 grams of carbohydrates, no more than 12 grams of fat and at least 3 units of vitamin Z.

   (a) Write down a system of linear inequalities which describe how many ounces of each food one can eat while satisfying the given requirements.
   (b) Graph the feasible set for the system of inequalities in part a.
   (c) Find all the corners of the feasible set you drew in part b.

8. One ounce of carrots contains 2 units of vitamin A, 1 unit of vitamin B and 2 units of vitamin C. One ounce of peas contains 1 unit of vitamin A, 3 units of vitamin B and 2 units of vitamin C. Suppose a dietitian wishes to serve a mixture of peas and carrots which contains a minimum of 4 units of vitamin A, 3 units of vitamin B and 3 units of vitamin C.

   (a) Write a system of linear inequalities which summarizes the above problem.

(b) Graph the feasible set.

9. Solve the linear program by the corner point method.

$$\begin{array}{lrcl} \text{min} & x+2y & & \\ \text{subject to} & x+y & \geq & 10 \\ & 3x+y & \geq & 19 \\ & x & \geq & 0 \\ & y & \geq & 0 \end{array}$$

10. Do the two problems on the Lindo instructions.

11. Consider the feasible set

$$\begin{array}{rcl} x+y & \leq & 10 \\ 2x+y & \leq & 12 \\ x & \geq & 0 \\ y & \geq & 0. \end{array}$$

(a) Sketch the feasible set.

(b) Give the augmented coordinates of each corner point of the feasible set.

(c) Give the remaining coordinates of the point where $x = 1$ and $s_1 = 7$. Is this point feasible?

12. Solve the following linear program by the simplex method.

$$\begin{array}{l} \max\ x+2y+z \\ \text{subject to } x+y+z \leq 20 \\ \qquad\qquad x+3y+2z \leq 90 \end{array}$$

13. Consider the linear program

$$\begin{array}{lrcl} \text{max} & x+y+z & & \\ \text{subject to} & x+y+2z & \leq & 10 \\ & 2x+y+4z & \leq & 10 \\ & x & \geq & 0 \\ & y & \geq & 0 \\ & z & \geq & 0 \end{array}$$

(a) Use Lindo to find the dual prices for the first two constraints.

(b) Suppose the right-hand side of the constraint $x + y + 2z \leq 10$ is decreased from 10 to 6. Use the dual price to find the new profit.

(c) Use Lindo to check your answer to part b).

14. (a) On two different Venn diagrams, shade the sets $(A \cup B) \cap C$ and $A \cup (B \cap C)$. In general, are these two sets the same?

    (b) Suppose that in a class of 60, 40 students passed the first exam, 35 students passed the second exam and 30 passed both exams. How many passed neither exam? How many passed at least one exam?

15. (a) Consider the letters A, B, C, and D.

        i. List the permutations of these letters taken two at a time.

        ii. List the combinations of these letters taken two at a time.

        Hint: You can use the formulas for $C(n,k)$ and $P(n,k)$ to see if you have the right number of entries on your list.

    (b) Suppose that we pick a letter from the alphabet and then choose a number from the set $\{1,2,3,4,5,6,7,8,9,10\}$. How many different ways can these choices be made? Explain how you arrived at your answer.

16. (a) Suppose that $P(A) = 0.5$, $P(B) = 0.6$, and $P(A \cap B) = 0.4$. Compute $P(B')$, $P(A \cup B)$ and $P((A \cup B)')$.

    (b) Suppose that you roll two die. What is the probability that the two die sum to 7?

17. At the first test, there were 107 students. Of these, 75 were men and 32 were women. There were 36 students wearing hats, but only two women with hats.

    (a) Compute the probability that a student is wearing a hat.

    (b) Compute the conditional probability of wearing a hat given that a student is male.

    (c) Compute the conditional probability of being male, given that a student is wearing a hat.

    (d) Are the events of wearing a hat and being male independent? Why?

18. Suppose that you flip a coin and then draw a number. If the coin comes up heads, then you draw from $\{1,2,3,4\}$ and if the coin is tails, then you draw from $\{1,2,3,4,5,6\}$.

    (a) What is the probability of drawing a six?

    (b) What is the probability of drawing a four?

    (c) If you draw a four, what is the probability that the coin was a head?

19. Suppose that a basketball player makes 90% of his free throws.

(a) What is the probability of making five out of ten free throws?

(b) Suppose that he shoots 10 free throws and receives a \$100 for each free throw that he makes over 7. (Thus, he receives nothing for 7 free throws, \$100 for 8, \$200 for 9 and \$300 for 10.) What is the expected value of this game?

20. Solve by the corner point method.

$$\begin{array}{lrcl} \text{max} & x + 2y & & \\ \text{subject to} & x + y & \geq & 3 \\ & 2x + 3y & \leq & 12 \\ & x & \geq & 0 \\ & y & \geq & 0 \end{array}$$

21. Solve the two problems given in the Lindo handout.

22. Consider the feasible set

$$\begin{array}{rcl} x + y & \leq & 5 \\ 2x + 3y & \leq & 12 \\ x & \geq & 0 \\ y & \geq & 0. \end{array}$$

Give all of the augmented coordinates for the points where (a) $x = 2$ and $y = 3$, (b) $x = 2$ and $s_1 = -2$, (c) $x = 7$ and $y = 5$. Which of the above points are feasible? Why?

23. Consider the tableau:

x	y	z	s_1	s_2	s_3	RHS
1	2	0	1	0	1	10
0	1	1	1	0	0	5
0	1	0	1	1	1	5

(a) Write the constraints as equations.

(b) Give the basic and nonbasic variables.

(c) Give the corner point associated to this tableau.

(d) If $y = 2$, $s_1 = 2$ and $s_3 = 0$, find the remaining augmented coordinates. Is this point feasible?

(e) Pivot so that the variable y becomes basic and the variable z becomes nonbasic. Find the new corner point.

24. Consider the simplex tableau:

P	x	y	s_1	s_2	RHS
1	0	0	-1	1	2
0	1	0	1	0	3
0	0	1	1	1	4

(a) Pivot so that the basic variables are P, x and s_1.

(b) The pivot you performed in part a) is not the pivot prescribed by the simplex method. Why not? Why is the tableau you obtained in part a) not allowed as a tableau in the simplex method?

25. (a) Solve the linear program by the graphical method.

$$\begin{array}{lrcl} \text{max} & x+y & & \\ \text{subject to} & x+2y & \leq & 4 \\ & 2x+y & \leq & 5 \\ & x & \geq & 0 \\ & y & \geq & 0 \end{array}$$

(b) Solve the linear program by the graphical method.

$$\begin{array}{lrcl} \text{max} & x+y & & \\ \text{subject to} & x+2y & \leq & 4 \\ & 2x+y & \leq & 6 \\ & x & \geq & 0 \\ & y & \geq & 0 \end{array}$$

(c) Determine the dual price of the second constraint in the linear program in part a).

26. (a) Shade the sets $(A \cup B)'$ and $A' \cap B'$ on two Venn diagrams. Are these two sets equal?

(b) How many ways can you rearrange the letters in 'LEWIS' if repetitions are not allowed?

(c) How many five letter words can you make from the letters in 'CLARK' if repetitions are allowed? (Hint: Parts b) and c) can be answered using the multiplication principle.)

27. Problem 12 is cancelled.

28. (a) Compute $C(5,2)$, $P(5,2)$ and $C(12,10)$. Your solution should indicate how you carried out the arithmetic. In particular, I do not want you to use a calculator for these computations.

(b) How many ways can you rearrange the letters in 'RUSSELLBROWN' using each letter only as often as it appears in Russell Brown's name? (For this part you will need a calculator.)

29. Let A and B be two events. Suppose that $P(A) = 0.3$, $P(B) = 0.4$ and $P(A \cap B) = 0.1$.

    (a) Compute $P(A \cup B)$.

    (b) Compute $P(A' \cap B')$.

30. (The due date on problem 15 has been changed from 11/15 to 11/17). A sample space contains 80 outcomes. Assume that all outcomes are equally likely. Let X and Y be two events in S and suppose that $n(X) = 50$, $n(Y) = 30$ and $n((X \cup Y)') = 20$.

    (a) Draw a Venn diagram summarizing this information.

    (b) Are the events X and Y independent?

    (c) Are the events X and X' independent?

31. Suppose that a pollster asks each respondent to roll a die and then answer a yes or no question. If the die is a one or two, then the person is answer yes if the last digit of their drivers license is odd and no otherwise. If the die is three through six, then the person is to answer the question "Do you like finite math?". Suppose that 40% of the people asked respond yes, what is the probability that one likes finite math.

32. An insurance company has found that a certain policy produces a claim of \$500 with probability of 0.05 and a claim of \$10,000 with probability of 0.001.

    (a) What should they charge for this policy in order to make a profit of \$10 per policy?

    (b) Which claims, on average, cost the company more: the small claims of \$500 or the large claims of \$10,000?

33. A company has developed a test for disease Y. Disease Y occurs in 70% of the patients to be tested. The test is accurrate in 80% of people with disease Y and in 75% of the people without disease Y.

    (a) Draw a probability tree which summarizes the above information.

    (b) If a person tests positive for disease Y, what is the probability that the person is ill with disease Y?

34. (a) An individual has \$500 to invest in stocks and bonds. The stock investment earns 10% and the bond investment earns 6%. If his total earnings are \$42, determine how he divided his money between stocks and bonds.

    (b) Give an example of a 2×2 system of equations with infinitely many solutions.

35. Find the solution by your favorite method.

$$\begin{array}{rrrcr} x & & -z & = & -1 \\ x & +2y & +2z & = & 6 \\ x & -4y & +z & = & 1. \end{array}$$

36. (a) A company manufacturers textbooks from cover board, paper and glue. One MA109 textbook uses 1.5 square feet of cover board, 135 sheets of paper and 1.5 ounces of glue. One MA123 textbook uses 2 square feet of cover board, 120 sheets of paper and 1 ounce of glue. An MA162 text book uses 2 square feet of cover board, 190 sheets of paper and 1.25 ounces of glue. Suppose that the company has 200 square yards of cover board, 20 reams of paper and 10 pints of glue. Write a system of equations whose solution tells how many of each book they should make in order to use all available materials.

    (b) Find the solution to the system

$$\begin{aligned} w + x + 2y + z &= 1 \\ 2w - x + 3y + 2z &= -1 \end{aligned}$$

when $x = 1$ and $z = -1$.

37. (a) Let $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ and suppose that $A^{-1} = \begin{pmatrix} 1 & 2 \\ 1 & -1 \end{pmatrix}$. Solve for x and y.

$$\begin{aligned} ax + by &= 1 \\ cx + dy &= -3 \end{aligned}$$

    (b) Find the inverse of

$$\begin{pmatrix} 1 & 0 & -2 \\ 2 & 1 & 0 \\ -4 & -2 & 1 \end{pmatrix}$$

38. Suppose that each ounce of food A has 2 grams of fat, 3 grams of carbohydrate and 2 units of Vitamin Z. Each ounce of food B has 3 grams of

fat, 1 gram of carbohydrates and 3 units of vitamin Z. Suppose that one needs to consume at least 6 grams of carbohydrates, no more than 12 grams of fat and at least 3 units of vitamin Z.

(a) Write down a system of linear inequalities which describe how many ounces of each food one can eat while satisfying the given requirements.

(b) Graph the feasible set for the system of inequalities in part a.

(c) Find all the corners of the feasible set you drew in part b.

39. Suppose that each ounce of food A has 2 grams of fat, 3 grams of carbohydrate and 2 units of Vitamin Z. Each ounce of food B has 3 grams of fat, 1 gram of carbohydrates and 3 units of vitamin Z. Suppose that one needs to consume at least 6 grams of carbohydrates, no more than 12 grams of fat and at least 3 units of vitamin Z.

    (a) Write down a system of linear inequalities which describe how many ounces of each food one can eat while satisfying the given requirements.

    (b) Graph the feasible set for the system of inequalities in part a.

    (c) Find all the corners of the feasible set you drew in part b.

40. Solve the linear program by the corner point method.

$$\begin{array}{lrcl} \text{min} & x+2y & & \\ \text{subject to} & x+y & \geq & 10 \\ & 2x+y & \geq & 14 \\ & x & \geq & 0 \\ & y & \geq & 0 \end{array}$$

41. Do the two problems on the Lindo instructions.

42. Consider the feasible set

$$\begin{array}{rcl} x+y & \leq & 5 \\ 2x+3y & \leq & 12 \\ x & \geq & 0 \\ y & \geq & 0. \end{array}$$

    Give all of the augmented coordinates for the points where (a) $x=2$ and $y=3$, (b) $x=2$ and $s_1=-2$, (c) $x=7$ and $y=5$. Which of the above points are feasible? Why?

43. Solve the linear program by the simplex method.

$$\begin{array}{lrcl} \max & x+y+2z & & \\ \text{subject to} & x+y+z & \leq & 4 \\ & 3x+z & \leq & 4 \\ & x & \geq & 0 \\ & y & \geq & 0 \\ & z & \geq & 0 \end{array}$$

44. Consider the linear program

$$\begin{array}{lrcl} \max & x+y+z & & \\ \text{subject to} & x+y+2z & \leq & 10 \\ & 2x+y+4z & \leq & 10 \\ & x & \geq & 0 \\ & y & \geq & 0 \\ & z & \geq & 0 \end{array}$$

(a) Use the Lindo printout below to answer the following questions. You should consider each question separately. That is after each question, assume that all coefficients and righthand sides are returned to their original values.

i. Suppose that righthand side of the constraint in row 2 is increased by 5. What is the new optimal objective function value?

ii. Suppose that the righthand side of the constraint in row 3 changed to 15. What is the new optimal objective function value?

iii. Suppose the coefficient of x increases to 5. What is the new optimal objective function value?

iv. Suppose the coefficient of y is increased by 4. What is the new optimal objective function value?

v. Suppose that we must choose between increasing the righthand side of row 2 by 3 or increasing the righthand side of row 3 by 3. Which increase will lead to a greater increase in the objective function value?

```
MAX     4 X + 5 Y
SUBJECT TO
       2)   2 X + 3 Y <=   15
       3)   4 X + 3 Y <=   20
END
```

```
LP OPTIMUM FOUND AT STEP      2

       OBJECTIVE FUNCTION VALUE

       1)      26.66667

 VARIABLE        VALUE          REDUCED COST
        X         2.500000          .000000
        Y         3.333333          .000000


      ROW   SLACK OR SURPLUS     DUAL PRICES
       2)          .000000         1.333333
       3)          .000000          .333333

NO. ITERATIONS=       2

RANGES IN WHICH THE BASIS IS UNCHANGED:

                           OBJ COEFFICIENT RANGES
VARIABLE         CURRENT        ALLOWABLE         ALLOWABLE
                  COEF          INCREASE          DECREASE
       X        4.000000        2.666667           .666667
       Y        5.000000        1.000000          2.000000

                           RIGHTHAND SIDE RANGES
     ROW         CURRENT        ALLOWABLE         ALLOWABLE
                  RHS           INCREASE          DECREASE
       2       15.000000        5.000000          5.000000
       3       20.000000        9.999999          5.000000
```

(b) Use Lindo to find the dual prices for the first two constraints.

(c) Suppose the right-hand side of the constraint $x + y + 2z \leq 10$ is decreased from 10 to 5. Use the dual price to find the new profit.

(d) Use Lindo to check your answer to part b).

45. Consider the linear program

$$\begin{array}{lrcl} \max & x + y + z & & \\ \text{subject to} & x + y + 2z & \leq & 10 \\ & 2x + y + 4z & \leq & 10 \\ & x & \geq & 0 \\ & y & \geq & 0 \\ & z & \geq & 0 \end{array}$$

(a) Use Lindo to find the dual prices for the first two constraints.

(b) Suppose the right-hand side of the constraint $x + y + 2z \leq 10$ is decreased from 10 to 5. Use the dual price to find the new profit.

(c) Use Lindo to check your answer to part b).

46. Use the Lindo printout below to answer the following questions. You should consider each question separately. That is after each question, assume that all coefficients and righthand sides are returned to their original values.

(a) Suppose that righthand side of the constraint in row 2 is increased by 5. What is the new optimal objective function value?

(b) Suppose that the righthand side of the constraint in row 3 changed to 15. What is the new optimal objective function value?

(c) Suppose the coefficient of x increases to 5. What is the new optimal objective function value?

(d) Suppose the coefficient of y is increased by 4. What is the new optimal objective function value?

(e) Suppose that we must choose between increasing the righthand side of row 2 by 3 or increasing the righthand side of row 3 by 3. Which increase will lead to a greater increase in the objective function value?

```
 MAX     4 X + 5 Y
 SUBJECT TO
        2)   2 X + 3 Y <=   15
        3)   4 X + 3 Y <=   20
 END

LP OPTIMUM FOUND AT STEP      2

       OBJECTIVE FUNCTION VALUE
```

```
       1)      26.66667

 VARIABLE        VALUE          REDUCED COST
        X         2.500000           .000000
        Y         3.333333           .000000


      ROW   SLACK OR SURPLUS     DUAL PRICES
       2)          .000000          1.333333
       3)          .000000           .333333

 NO. ITERATIONS=       2

 RANGES IN WHICH THE BASIS IS UNCHANGED:

                           OBJ COEFFICIENT RANGES
 VARIABLE         CURRENT        ALLOWABLE         ALLOWABLE
                   COEF          INCREASE          DECREASE
        X        4.000000         2.666667           .666667
        Y        5.000000         1.000000          2.000000

                           RIGHTHAND SIDE RANGES
      ROW         CURRENT        ALLOWABLE         ALLOWABLE
                    RHS          INCREASE          DECREASE
        2       15.000000         5.000000          5.000000
        3       20.000000         9.999999          5.000000
```

(f) i. On separate Venn diagrams, shade the sets $(A \cup B) \cap C'$ and $(A' \cap B') \cup C$.

ii. Suppose that A and B are subsets of a universe U. If $n(U) = 50$, $n(A) = 20$, $n(B) = 40$ and $A' \cap B' = \emptyset$, then compute $n(A \cap B)$.

(g) i. Compute $C(12, 4)$ and $P(12, 4)$.

ii. Consider the set of letters $\{A, B, C, D\}$.

A. List all the combinations of these letters taken three at a time.

B. List all of the permutations of these letters taken two at a time.

(h) i. Draw the sets $A \cap (B \cup C)$ and $(A \cap B) \cup C$ on separate Venn diagrams. In general, are these sets equal?

ii. Suppose that in a class, 40 students understand the simplex method, 50 students understand the corner point method and 30 students understand both. If every student understands either the simplex method or the corner point method, how many students are in the class? Use a Venn diagram to illustrate your answer.

(i) i. How many permutations are there of the letters in SIX? List them.

 ii. How many combinations are there of the letters in HARE taken two at a time? List them.

 iii. How many combinations are there of the letters in HARE taken three at a time? List them.

(j) Suppose that we flip 4 coins and think of the outcomes as 4 letter words using the letters H and T.

 i. How many outcomes are possible?

 ii. How many outcomes are all heads?

 iii. How many outcomes have at least one tail? Why?

 iv. How many outcomes have exactly two tails? List them.

 v. How many outcomes have exactly three tails? List them.

Suppose that you roll two dice.

1. List the sample space for this experiment.
2. Compute the probability of obtaining at least one 6.
3. Compute the probability of obtaining exactly one 6.
4. Compute the probability of obtaining no sixes.
5. Compute the probability that the numbers on the two dice sum to 4.

1. On two separate Venn diagrams, shade $A' \cap B$ and $(A \cup B)'$. Are these sets the same?
2. If $A = \{1, 2, 4, 8\}$ and $B = \{1, 3, 5, 7\}$, list the elements of $A \cup B$ and $A \cap B$.

1. List all the combinations with two elements chosen from the letters in QUIT.
2. List all the permutations with two elements chosen from the letters in QUIT.
3. Compute $P(4, 2)$ and $C(4, 2)$ and check to see if your lists in parts a and b have the correct number of elements.

Suppose that A and B are events with $P(A) = P(B) = 0.5$ and $P(A \cup B) = 0.8$.

1. What is $P(A \cap B)$?
2. What is $P(A')$?
3. Are the events A and B independent? As always, you should explain how you arrived at your answer.

Suppose that you flip 4 coins and will receive a payment of \$2 multiplied by the number of heads you obtain.

1. Compute the probabilities of obtaining 0, 1, 2, 3 or 4 heads.
2. Why should your answers in a) sum to one?
3. What is the expected value of playing this game?

A company has developed a test for disease Y. Disease Y occurs in 70% of the patients to be tested. The test is accurate in 85% of the people with disease Y and 70% of the people without disease Y.

1. Draw a probability tree which summarizes the above information.
2. If a person tests positive for disease Y, what is the probability that the person is ill with disease Y?

An individual has \$600 invested in stocks and bonds. The stock investment earns 10% and the bond investment earns 6%. If his total earnings are \$44, determine how he divided his money between stocks and bonds.

Give an example of a 2×2 system of equations with infinitely many solutions.

At the Smith family's favorite fast food restaurant, a hamburger contains 10 grams of fat, 70 grams of carbohydrates and 20 grams of protein. French fries contain 40 grams of fat, 50 grams of carbohydrates and 5 grams of protein. Milk shakes contain 60 grams of fat, 5 grams of carbohydrates and 15 grams of protein. Suppose that the Smiths have 200 grams of fat, 600 grams of carbohydrates and 150 grams of protein at lunch. Write down a system of equations whose solutions are the number french fries, milk shakes and hamburgers that the family had for dinner. Be sure to indicate what each variable in your system stands for. DO NOT solve the resulting system of equations.

1. Solve the system of equations

$$\begin{array}{cccl} x & +y & +z & = 6 \\ & y & +z & = 5 \\ x & +y & & = 3 \end{array}$$

2. Solve the system of equations:

$$\begin{array}{cccl} x & +y & +z & = 4 \\ & y & +z & = 1 \\ x & +y & & = 5 \end{array}$$

1. Find two solutions to the system

$$\begin{aligned} w + x + 2y + z &= 4 \\ w + 2x - y + z &= 4. \end{aligned}$$

1. Find the inverse of the matrix

$$\begin{pmatrix} 1 & 1 & 0 \\ 1 & 2 & 0 \\ 1 & 1 & 1 \end{pmatrix}$$

2. Use the inverse in part a) to solve the system

$$\begin{aligned} x + y &= 1 \\ x + 2y &= 2 \\ x + y + z &= 3 \end{aligned}$$

Use LINDO to find the solutions to the following problems. Turn in your LINDO printout along with your solutions.

1. You want to start a business which will make tables, chairs and desks. You have only 100 units of wood available and you know that it takes 6 units of wood to make a table, 4 units to make a chair and 5 units of wood to make a desk. You also have the constraint that you only have 600 hours of labor available to you. It takes 10 hours to make a table, 15 hours to make a chair and 20 hours to make a desk.

   If you can earn a profit of \$60 per table, \$50 per chair and \$40 per desk, how many of each should you produce to maximize your profit? How much profit will you make?

2. We change our process so that now a chair will require 5 units of wood, but the desks will only take 2 units of wood. This new type of chair will now take 35 hours to build. All else remains the same.

   How many of each product should we make now? What is our profit?

1. Graph the feasible set: $x + y \leq 9$
   $x \geq 2y$
   $x \geq 0, \; y \geq 0$

2. Consider the tableau:

x	y	z	s_1	s_2	s_3	RHS
1	2	0	1	0	1	10
0	1	1	1	0	0	5
0	1	0	1	1	1	5

(a) Write the constraints as equations.

(b) Give the basic and nonbasic variables.

(c) Give the corner point associated to this tableau.

(d) If $x = 2$, $z = 2$ and $s_2 = 0$, find the remaining augmented coordinates. Is this point feasible?

(e) Pivot so that the variable y becomes basic and the variable z becomes nonbasic. Find the new corner point.

3. Consider the feasible set

$$\begin{aligned} x + y + z &\leq 10 \\ x &\leq 5 \\ 2x + y + z &\leq 12 \\ x \geq 0, y \geq 0, z \geq 0 \end{aligned}$$

(a) Introduce the augmented coordinates for this feasible set and give the equations relating the decision variables x, y and z to the slack variables s_1, s_2 and s_3.

(b) Suppose $x = 0$, $y = 0$ and $z = 0$. Give the values of the slack variables.

(c) Suppose$x = 4$, $y = 5$ and $z = 6$. Give the values of the slack variables. Is this point feasible? As always, you should justify your answer.

(d) Suppose the slack variable for the first constraint is 10 and $x = 5$ and $y = 5$. Find the values of the remaining augmented coordinates. Is this point feasible? Why?

4. Solve the linear program: max $3x + y$
Subject to
$x + y \leq 9$
$x \geq 2y$
$x \geq 0, \ y \geq 0$

A mining company has two mines whose production is given the table below.

	Lead	Tin	Platinum
Mine 1	6 lbs/day	3 lbs/day	1 lb/day
Mine 2	7 lbs/day	4 lbs/day	2 lbs/day

Mine 1 costs \$20,000 a day to operate while Mine 2 costs \$15,000 per day. In order meet their orders they need to produce at least 700 lbs of lead, 900 lbs of tin and 150 lbs of platinum. Write down a linear program whose solution gives the number of days the company should operate each mine while minimizing its costs. Do not solve the linear program.

Consider the following simplex tableau:

Row	P	x_1	x_2	x_3	s_1	s_2	s_3	*RHS*
0	1	0	−1	−2	0	0	0	8
1	0	1	2	2	0	0	2	8
2	0	0	1	1	0	1	1	2
3	0	0	3	2	1	0	7	6

1. List the basic variables____________
2. List the nonbasic variables ____________
3. Give the augmented coordinates of the corner associated to this

   tableau. ____________
4. Give the variable at the top of the pivot column. ____________
5. Give the number of the pivot row. ____________
6. Give the current value of the objective function. ____________
7. Is this value the optimal value? Why or why not?

Consider the tableau:

P	x_1	x_2	s_1	s_2	*RHS*
1	−1	1	0	0	2
0	1	2	0	1	2
0	1	3	1	0	1

1. According to the simplex method, which pivot should you perform. (That is, what will the new basic variables be after you pivot?)
2. Pivot so that x_1 and s_1 are the new basic variables. (Note that this may not be the pivot prescribed by the simplex method.)
3. Does the new tableau correspond to a corner point of the feasible set? Why or why not?

How many three letter (sensible or nonsensible) words are there which are of the form consonant, vowel, consonant. Please explain your reasoning.

Is it true that $(A \cap B) \cup C = A \cap (B \cup C)$? Draw a Venn diagram to justify your answer. Unsupported answers will receive no credit.

There are 5 people in the Smith family. Use the multiplication principle to explain why there are 120 ways that all of the members of the Smith family may stand in a line for a family photograph.

Answer the following. You must write out your answer using factorials or writing out which integers you will multiply to obtain the answer. You do not need to give the numerical answer.

1. How many different 4 letter words can you form using the letters 'ABCDEFG'?. Assume that repetitions are not allowed.
2. How many distinguishable permutations are there of the letters in 'BOZOZ'?
3. How many permutations are there of the letters in 'COURANT'?

A die is rolled and a coin is flipped. You are given \$1 if the coin lands heads up. You are also given \$2 if the die shows an odd number. What is the probability of winning at least \$1 when playing this game one time?

A coin is flipped five times. What is probability of obtaining at least one head?

You are to draw two balls without replacement from an urn which contains 5 red and 4 blue balls.

1. Draw a tree which summarizes this game.
2. Compute the probability that the second ball is red.
3. Compute the conditional probability that the first ball is blue given that the second ball is red.

You play a game where you are to roll a die 5 times. If you obtain 5 sixes, you will receive \$3000 and if you obtain exactly 4 sixes you will receive \$300. What is the expected value of this game? Would it be profitable to play this game if you have to pay \$1.50 for each play?