MA202:004 Problem set 5

March 4, 1999

Below are a list of problems taken from the material in Chapter 4 and Chapter 6. We will discuss a few topics from Chapter 6 as we work on these problems.

1. (This problem is not for discussion, you should imitate the arguments we gave in problem #13 of section 1.3.) Suppose we have a board with 32 squares arranged in a 2×16 rectangular array. We want to place one domino which is 2×1 on the board so that it lines up with the squares on the board. How many places can we put the domino. (Hint: Try some smaller cases–2×1, 2×2 and so on. Do you see a pattern? As always, the best papers will try to explain why this pattern arises. To do this you should ask how many new positions arise when we go from a $2 \times n$ board to a $2 \times n + 1$ board.)

2. (Section 4.3 #20) The front wheel of a tricycle has a circumference of 54 inches and the back wheel have a circumference of 36 inches. If points P on the front wheel and Q on the back wheel are touching the sidewalk when Marja starts to ride, how far will she have to ride before they are both touching again. How many times will each of the wheels turned around when P and Q first return to the ground? (Hint: Try some smaller cases–what if one circle is of circumference 2 and the other is of circumference 3. What if one is of circumference 4 and the other of circumference 6?)

3. One way to test if a whole number is divisible by three is to sum the digits and then the number is divisible by three exactly when the sum of the digits is divisible by three. For example, for 121, the sum of the digits is $1 + 2 + 1 = 4$ which is not divisible by 3. Hence, 121 is not divisible by 3. If we try 123, then $1 + 2 + 3 = 6$ which is divisible by 3, and $123 = 3 \times 41$ is divisible by 3.

   Read about this test in page 265.

   Imitate the example on page 265 to show that 42,432 is a multiple of 3. What will the remainder be when 42,432 is divided by 9? Again, you should use the ideas on page 265 to find the remainder.

   Of course, you may check your work using a calculator.

4. (The clock, again.)

   (a) Consider an ordinary clock. How far (in minute marks) does the hour hand travel in one hour? How far does the hour hand travel in one minute?

   (b) At T minutes after 1 pm, where (in minute marks) is the hour hand pointing?

(c) Write down and solve an equation for the first time after 1 pm when the minute and hour hand are in the same place.

(d) How many times does the minute hand pass the hour hand in a twelve hour period? (Be careful, if you think about the period from 12 noon to 12 midnight, we only want to count the instant when both hands are at the 12 once. This way the answer is the same for all twelve hour periods.) Can you use this to give a different explanation for your answer to the question in c)?

5. A soccer ball is in a 1/3 off sale and the sale price is \$6. What was the original price? Give two ways of explaining your answer–one using algebra and one using an argument that does not rely on algebra.

6. Below is 1/4 of a figure. Draw the whole figure. Does this problem have exactly one correct answer? (This problem is one activity suggested in the NCTM standards, K–4 standard 12, fractions and decimals.)

7. Show that $9/100 < 10/101$. Do this using your calculator and by obtaining a common denominator. (See page 381 for a discussion of comparing fractions or rational numbers.) Can you think of a third way to do this?

8. For two fractions a/b and c/d where a, b, c, d are integers and b and d are both natural numbers, define the mediant by

$$\frac{a}{b} \oplus \frac{c}{d} = \frac{a+c}{b+d}.$$

(a) Show that if $a/b < c/d$, then

$$\frac{a}{b} < \frac{a}{b} \oplus \frac{c}{d} < \frac{c}{d}.$$

(b) Is

$$\frac{a}{b} + \frac{c}{d} = \frac{a}{b} \oplus \frac{c}{d}.$$

always true?

(c) Is

$$\frac{a}{b} + \frac{c}{d} = \frac{a}{b} \oplus \frac{c}{d}.$$

ever true?