MA202:004 Problem set 6

March 3, 1999

We have a test on Tuesday, March 9 1999. This test will cover all problems discussed since the previous test. We will cover all of set 5, all of set 4 and items 3 and 4 on set 3. There were several difficult problems on these sets. In order to improve our performance on the next exam, please work the following review problems. For those who have done well on the previous sets, these will help cement ideas. For those who are struggling, these problems should provide a second chance. We will discuss these problems next week and hand in solutions on Thursday, March 4. Your graded solutions will be available at my office Friday morning so that you may use them to study for the test.

1. If today is Thursday, what day of the week will it be in 5^{222} days?

2. (20 points) Consider the following Fibonacci-like sequence, $G_1, G_2, \ldots$, which is defined by $G_1 = 2$, $G_2 = 6$ and then $G_n = G_{n-1} + G_{n-2}$ for $n = 1, 2, 3 \ldots$. Thus the next number is the sequence is $G_3 = G_1 + G_2 = 8$.

   (a) Give the first ten numbers in the sequence, G_1 through G_{10}.

   (b) Which G_n are even? Why?

   (c) Which of G_1 through G_{10} are divisible by 3? Make a prediction as to the first number G_k with $k \geq 11$ which will be a multiple of 3. Check if you are correct.

   (d) What is the GCD of consecutive pairs of numbers from this sequence? Use the Euclidean algorithm to explain why your answer is correct.

3. Consider a $2 \times n$ board, again. How many ways can we place an L-shaped domino on the board so that the squares on the domino line up with the edges on the board? Try this for small boards. See if a pattern develops. Explain why your pattern holds. Use this pattern to determine the number of ways that we can cover a 2×19 board.