MA202:004 Problem set 7

March 30, 1999

Below is a series of problems on probability. We will begin work on Thursday 3/11/99 and discuss them on Tuesday, 3/23/99. They will be due on Thursday, 3/25/99. These problems relate to probability. You should skim the Chapter on probability, Chapter 9. Pay special attention to Examples 9.11 and 9.22.

Set 5

8. For two fractions a/b and c/d where a, b, c, d are integers and b and d are both natural numbers, define the mediant by
$$\frac{a}{b} \oplus \frac{c}{d} = \frac{a+c}{b+d}.$$
   (a) Show that if $a/b < c/d$, then
   $$\frac{a}{b} < \frac{a}{b} \oplus \frac{c}{d} < \frac{c}{d}.$$
   (b) Is
   $$\frac{a}{b} + \frac{c}{d} = \frac{a}{b} \oplus \frac{c}{d}.$$
   always true?
   (c) Is
   $$\frac{a}{b} + \frac{c}{d} = \frac{a}{b} \oplus \frac{c}{d}.$$
   ever true?

Set 7

1. Roll a pair of dice at least one hundred times and record the sum of the digits on each roll. Use this information to answer the following questions.
   (a) Which sum occurs most often? Do different sums appear to be equally likely?
   (b) What is the empirical probability of obtaining a sum of 6?
   (c) If you roll the dice 31,000 times, how often do you expect to obtain a sum of 6?
2. (a) If we roll a pair of dice and record the pair of numbers we obtain, how many different pairs are possible?
   (b) How many of these pairs of numbers add to give a sum of 6?
   (c) We expect that each of the pairs listed in part a) is equally likely. Compute the theoretical probability of obtaining a 6. Compare your answer with the empirical probability in the previous problem. Are you close? Do you expect to obtain exactly the same number?

(d) Compute the theoretical probability of obtaining a 7 and compare it with the empirical probability obtained using the information in the previous problem.

3. Suppose we roll one 12 sided die and one 4 sided die. The twelve sided die is numbered 1 through 12. The four sided die is numbered 1 through 4. What is the probability that we obtain a sum of 7? What is the probability we obtain a sum of 17?