MA202:004 Problem set 8

April 13, 1999

Below are solutions to homework set 8.

1. (a) For our experiment, roll a pair of dice a large number of times. Record the number of rolls, the number of times we obtain the outcome 1 and 1 and the number of times we obtain the outcome 2 and 1 (in any order).

   Our earlier experiment where we recorded the sum of the numbers on each die provides us with the needed information. There, we obtained the sum of 2, which can only happen if we obtain 1 on each die, 5 times. The sum of 3, which happens if we obtain 2 on one die and 1 on the other die, occurred 12 times. The total number of rolls was 226.

   Thus, the empirical probability of obtaining a 1 on both dice is $5/226 \approx 0.022$. The empirical probability of obtaining a 1 on one die and and 2 on the other die is $12/226 \approx 0.053$. It appears that these two outcomes are not equally likely.

   (b) We should not use the 21 unordered pairs as our sample space in problem 2 on set 7, since these outcomes are not equally likely. Our experiment indicates that the outcome of (1,1) and the outcome of 2 and 1 in any order are not equally likely.

2. The total number of outcomes is $1200 = 5 \times 12 \times 20$. To see this, first think about rolling a 5 sided die and a 12 sided die. This will give sixty possibilities as we can see by imagining a 5 by 12 rectangle where each box corresponds to one pair of outcomes. Next, if we imagine 20 by 60 rectangle, where the 20 spaces along the side of length 20 corresponds to the sides on the 20 sided die and each of the 60 spaces along the side of length 60 corresponds to one of the 60 pairs coming from the 5 and 12 sided dice. This 60 by 20 rectangle has 1200 boxes which give the 1200 possibilities.

   A simpler (better?) solution is to observe that if we write down the outcome of the 5 sided die, the 12 sided die and the 20 sided die, we have 5 choices for the first number, 12 choices for the second number and 20 choices for the third number. Multiplying these together gives us a total of 1200 outcomes altogether.

   Remark: The Third principle of counting, page 571, states that the number of ways that A and B occur is the number of ways that B occurs multiplied by the number of ways that A occurs, given that B has occurred. This is the principle that we are using when we multiply 5×12 to give the number of outcomes from rolling the two dice.

3. We have that $(n+1)! = (n+1)n!$. Thus, $n!$ is a divisor of $(n+1)!$. This implies that $GCD(n!, (n+1)!) = n!$.

4. To list the permutations of FACT, first list all the permutations which begin with F, then those that start with A, then with C and then with T. To construct the permutations that start with F, we need to know all permutations of ACT. Here, we can list those starting with A, then with C then with T. Now that we have fixed the first two letters, it is easy to write out the two permutations of the remaining pair of letters. In a similar way, we list the permutations which begin with A, C and T. Following this procedure gives us the list:

FACT	AFCT	CFAT	TFAC
FATC	AFTC	CFTA	TFCA
FCAT	ACFT	CAFT	TAFC
FCTA	ACTF	CATF	TACF
FTAC	ATFC	CTFA	TCFA
FTCA	ATCF	CTAF	TCAF

   We can check if we are done by observing that we have found 24=4! permutations and we know from the text that this is the correct number of permutations.

5. To form a permutation of 16 letters, we may choose the first letter and then follow this with one of the P permutations of the remaining 15 letters. This gives a total of $16P$ possible permutations. One can imagine a rectangle with dimensions 16 by P as in problem 2 above.

   Remark: Many students answered this question by observing that $P = 15!$ and the number of permutations of 16 letters is $16! = 16P$. This is a correct solution and you received full credit. However, I had hoped that students would realize that one could determine the answer without knowing the value of P. In fact, this type of argument is needed to explain why $P = 15!$.

6. After we have chosen a door, there are two doors remaining. Since only one door is in front of a Chevy Blazer, there will always be at least one door which contains potato chips.

   Let us consider two strategies. The "switch" strategy where we will always switch to the remaining unopened door after Monty shows us a door with \$50 worth of potato chips and the "no-switch" strategy where we stick with our original choice.

   With the no-switch strategy, our probability of winning the Blazer is 1/3 since that is the probability of picking the door in front of the Blazer in our initial pick. With the switch strategy, our probability of losing is 1/3, since we lose exactly when we pick the Blazer in our initial choice and then switch to another door. Thus our probability of winning is 2/3.

   Since our probability of winning is 2/3 with the switch strategy, and only 1/3 with the no-switch strategy, it is in our best interest to use the switch strategy. Over a large number of plays, the switch strategy will win the Chevy Blazer most often.

Remark: Several papers mentioned another strategy where we choose at random after one door has been opened. This strategy has a probability of 1/2 of winning. Since the "choose at random" strategy will involve switching half the time, it is plausible to believe that this strategy will be successful more often than the no-switch strategy and less often than the switch strategy.

Exercise: For practice, you should try computing the probabilities of winning with various strategies if there are 4 doors or 400 doors.