MA202:004 Problem set 9
April 15, 1999 Solutions

1. Let us pick two towns, which we call A and B. We let $D(A)$ denote the set of all towns which we can travel to from A, possible passing through other towns along the way. Similarly, we let $D(B)$ be the set of all towns we can travel to from B. By our assumption that A is connected to at least 7 other towns, we know that $D(A)$ contains at least 8 towns, A and the seven towns that are connected to A. For the same reason, the set $D(B)$ contains at least 8 towns. Since there are only 15 towns, the two sets $D(B)$ and $D(A)$ must have at least one town in common, call it C. Thus we can drive from A to B by passing through C.

   Remarks: A number of good questions were raised by students as we worked on this problem. One was whether each town was connected directly to seven other towns or if the road was allowed to pass through other towns along the way. For this question, it doesn't matter. Another, question raised was what happens if the roads are one-way roads. We assume the roads are two-way roads. I am not sure what happens if we have one-way roads.

2. (a) Each of the seven teams plays 10 games. Thus there are 70 team listed on the schedule. However, each game takes two teams, so there are only 35 games played.

   (b) If we try to prepare such a schedule, we find that each team in the four team division plays 6 games in its division and needs 4 games out of the division, thus the four team division must play 16 games against the three team division. Each team in the three team division plays 4 games in its division and thus needs 6 games out of the division. Thus the three team division must play a total of 18 games against the four team division.

   Since the number of out-of-division games for each is division is different, no such schedule is possible.

   Exercise: Can you replace ten by some other total of games so that the number of out of division games is the same for each division?

   Suppose we have four teams in each division. Each team plays exactly 10 games and each team plays two games against every team in their division. How many games are played between the two divisions?

3. If there are an odd number of teams in each league, then each day we must have at least two teams idle or we must have at least two teams engaged in inter-league play. Neither is practical for major league baseball. In the current schedule all teams play every weekend and inter-league play occurs only during two periods in June and July.

Remark: The NBA has an odd number of teams, 29. I expect this is practical in basketball because basketball teams do not play as frequently as baseball teams.

Exercise: Tuesday in class, I suggested that the 29 teams in the National Basketball Association play 81 game schedules. Is this possible? (Hint: How many games would be played altogether?)

4. There are 30 teams and each plays 162 games. So a total of $30 \times 162 = 4860$ teams are listed on the schedule. Each game requires two teams, so there are only $4860/2 = 2430$ games played.

5. The pictures below suggest that the information in the following table is correct:

Number of sides	Sum of angles
3	$180°$
4	$360°$
5	$540°$
6	$720°$
7	$900°$
n	$(n-2)180°$

This table also gives our guess for the angle sum of an n-gon. Our examples indicate that an n-gon can be divided in $n-2$ triangles so the sum of the interior angles is $n-2$ times sum of the angles for a triangle.

Exercise: Can you draw an n-gon that cannot be divided into $n-2$ triangles?

6. The total of the angles in a hexagon is $720°$. If the hexagon is regular, then each angle is $720°/6 = 120°$. The total of the angles in a pentagon is $540°$ and thus each angle is $540°/5 = 108°$. If the vertices of one regular pentagon and two regular hexagons meet at a point, then the sum of the angles is only $348°$. Thus, these shapes do not completely fill up the plane since the sum is less than $360°$.

Remark: This pattern is possible on the surface of a sphere because the sum of the interior angles of a spherical n-gon is larger than the sum of the interior angles of a planar n-gon.