Mathematical problem solving — Russell Brown

Test 1 — 9 February 1999

Answer all of the questions on the answer sheets provided. Provide as much explanation as you can. You may use a calculator.

1. (10 points) State the pigeonhole principle and write a sentence to explain why it is true.
2. (10 points) Suppose that we have 2 blue socks, 13 red socks and 12 orange socks. How many must we draw in order to be sure that we have a three socks of the same color?
3. (20 points) We have nine tiles which are equilateral triangles and each side is one unit long. You are to arrange the triangles on a table so that they touch only along an entire edge and so that each triangle touches at least one other triangle.
   (a) What is the largest perimeter possible?
   (b) Is a perimeter of 10 possible?
4. (10 points) Illustrate Gauss's trick by using it to compute the sum
$$1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10.$$
5. (10 points) Compute the sum of first 99 odd numbers. (The first odd number is 1.)
6. (10 points) Suppose that we have 6 A's and 6 B's arranged as indicated:
$$AAAAAABBBBBB$$
Switching just two adjacent letters, what is the least number of moves you can make to achieve the arrangement below?
$$BABABABABABA$$