Mathematical problem solving — Russell Brown

Test 2 — 9 March 1999

Answer all of the questions on the answer sheets provided. Provide as much explanation as you can. You may use a calculator.
Note that one question is partly extra credit. However, no paper will receive more than 70 points.

1. (10 points) What is the largest set of whole numbers for which no difference of two of the numbers is a multiple of 11?

2. (10 points) Consider the sequence of Fibonacci-like numbers H_n where $H_1 = 12$ and $H_2 = 9$ and the remaining numbers are given by the formula $H_n = H_{n-1} + H_{n-2}$ for $n = 3, 4$ and so on. Thus, $H_3 = 21$.

   (a) Find H_1 through H_6.

   (b) What is $GCD(H_{100}, H_{101})$?

3. (10 points) The table below allows us to convert time in minutes into time in seconds. Fill in the missing entries in the table and then write a clear and simple rule which explains how to go from a number in the right-hand column to a number in the left-hand column.

Minutes	Seconds
1	60
2	
	30
0.6	
	300

4. (10 points) What is the remainder when 1,000,000,000,200,000,000,000,007 is divided by 9?

5. (10 points) A red soccer ball with blue markings is on sale for 1/4 off. If the sale price is \$9.99, what is the original price?

6. (10 points) What is the remainder when 2^{1000} is divided by 7?

7. (10 points + 5 points extra credit) Consider the 3×1 dominoes drawn below. How many ways can you exactly cover a $3 \times n$ board with these dominoes?

   (a) Begin by considering smaller boards and complete the following table.

Size of board	Coverings
3×1	1
3×2	1
3×3	
3×4	
3×5	
3×6	

   (b) Do you see a pattern? (You may want to try a 3×7 board or larger ones.) Explain what the pattern is and use it to tell me how many ways that we can exactly cover a 3×11 board.

   (c) Explain why the pattern you found in part b) arises.