Prof. Readdy
Review II

Very Brief Answers

1. -3

2a. 4
 b. $(-4)^3$
 c. $-1/32$

3. a. Yes
 b. $\mathrm{Diag}\{1,1,322\}$
 c. Pick any 3x3 matrix C with $|C| = \sqrt{2}$

4. a. $x_1 = -2/3$, $x_2 = 1/3$, $x_3 = 2/3$, $x_4 = 1/3$
 b. 14
 c. 0

5 a. FALSE. Let $A = \mathrm{diag}\{2, 1, ..., 1\}$ and $B = \mathrm{diag}\{3,1, ..., 1\}$.
 Then $A+B = \mathrm{diag}\{5,2,..., 2\}$ and $\det(A+B) = 5*(2^{n-1})$.

b. TRUE. (Can you prove it?)

c. TRUE. (Can you prove it?)

d. FALSE. If $\det A = 0$ (i.e., A not invertible), then you are stuck.

6. a. Carefully row-reduce (or column reduce) to show this.
 b. See class notes.

7. a. IS a subspace
 b. NOT a subspace
 c. IS a subspace
 d. NOT a subspace
 e. IS a subspace

8. a. 2
 b. 5
 c. 5
 d. 2, 2, 3

9. a.

$$A \sim \dots \sim \begin{bmatrix} 1 & 0 & -2 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

So basis for Col(A) is {columns 1, 2 & 4 of A).

Basis for Row(A) is {rows 1, 2, & 3 of row-reduced A}
Basis for Nul(A) is $[2,-1,1,0]^T$.

b. i. Check $T(A + B) = T(A) + T(B)$ and $T(cA) = c\, T(A)$ (Easy)
ii. Range(T) = all 2x2 matrices with real entries
iii. ker(T) is the zero 2x2 matrix.

c. For a. $\{t, t^2, .., t^n\}$
For c. Assuming the vectors a and b are nonzero and linearly independent, then basis is {a,b}. If a and b are dependent and a is nonzero, then {a}. If both are zero, then no basis.
For e, take the matrices $\{E_1, ..., E_n\}$ with zeroes everywhere except for a 1 in the (i,i) entry of the matrix E_i.

10. You should be able to come up with many reasons...

11. a. $5(1+2t^2) -2(4+t+5t^2) +1(3+2t) = 0$.
b. Let's row-reduce the basis vectors and solve for the coordinates at the same time:

$a(1-t^2) + b(t-t^2) + c(2-t+t^2) = 1 + 3t - 6t^2$:

becomes

$$\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & -1 \\ -1 & -1 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ -6 \end{bmatrix}$$

Row-reduce:

$$\left[\begin{array}{ccc|c} 1 & 0 & 2 & 1 \\ 0 & 1 & -1 & 3 \\ -1 & -1 & 1 & -6 \end{array}\right] \sim \ \ldots \left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -1 \end{array}\right]$$

Note the columns have 3 pivots, so they form a basis for P_2.
The coordinates are $[3\ 2\ -1]^T$.

$[1\ 2\ 3]^T$ in the basis B is $7-t$ in the standard basis.

There are many ways to do this!

12. a. a is n; b is not; c depends on the vectors -- could be 0, 1 or 2; d is not; e is n.

b. If A has 0 pivots, then nullity = 5;
If A has 1 pivot, then nullity = 4;
If A has 2 pivots, then nullity = 3;
If A has 3 pivots, then nullity = 2.

Examples are omitted.

13. a. Row reduce

$$[\; B \mid A \;] \text{ to } [I \mid B^{-1} A]$$

where

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 1 & 2 \\ 0 & -1 & -4 \\ 0 & 0 & 1 \end{bmatrix}$$

b. Multiply $B^{-1} A \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$