

PROFESSOR READDY'S OFFICIAL GRAHAM-SCHMIDT CHEAT SHEET

Given $W = \text{span}\{\vec{v}_1, \dots, \vec{v}_k\}$, Gram-Schmidt will give an orthonormal basis $\{\vec{u}_1, \vec{u}_2, \dots\}$ for W :

Let

$$\begin{aligned}\vec{w}_1 &= \vec{v}_1 \\ \vec{w}_2 &= \vec{v}_2 - \text{proj}_{\vec{w}_1} \vec{v}_2 \\ \vec{w}_3 &= \vec{v}_3 - \text{proj}_{\vec{w}_1} \vec{v}_3 - \text{proj}_{\vec{w}_2} \vec{v}_3 \\ \vec{w}_4 &= \vec{v}_4 - \text{proj}_{\vec{w}_1} \vec{v}_4 - \text{proj}_{\vec{w}_2} \vec{v}_4 - \text{proj}_{\vec{w}_3} \vec{v}_4 \\ &\vdots\end{aligned}$$

where

$$\text{proj}_{\vec{w}} \vec{v} = \frac{\vec{v} \cdot \vec{w}}{\vec{w} \cdot \vec{w}} \vec{w}$$

Finally, let

$$\begin{aligned}\vec{u}_1 &= \frac{\vec{w}_1}{|\vec{w}_1|} \\ \vec{u}_2 &= \frac{\vec{w}_2}{|\vec{w}_2|} \\ \vec{u}_3 &= \frac{\vec{w}_3}{|\vec{w}_3|}\end{aligned}$$

etc.