Name: Prof. Readdy Student ID: Answer Key, (version C1,

Section: _____

Do not remove this answer page. You will return the whole exam. You will be allowed two hours to complete this test. No books or notes may be used. You may use a graphing calculator during the exam, but NO calculator with a Computer Algebra System (CAS) or communication capabilities is permitted. You may not use a phone or other communication device during the exam.

The exam consists of 12 multiple choice questions that are worth 5 points each and 4 free response questions that are worth 10 points each. You should work the multiple choice questions on the question page. After you have checked your work carefully, record your answers by completely filling in the circle below that corresponds to your answer. If you must change your answer, make a note on the front of the exam. Be sure to check carefully when you transfer your answers to the cover sheet.

Show all work to receive full credit on the free response problems. You do not need to compute a decimal approximation to your answer. For example, the answer 4π is preferred to 12.57.

Multiple Choice Questions

1	A	B	**C**	D	E	**7**	**A**	B	C	D	E
2	A	B	**C**	D	E	**8**	A	B	**C**	D	E
3	A	B	C	**D**	E	**9**	A	B	C	**D**	E
4	A	B	C	D	**E**	**10**	A	**B**	C	D	E
5	A	B	C	**D**	E	**11**	**A**	B	C	D	E
6	A	**B**	C	D	E	**12**	**A**	B	C	D	E

SCORE

Multiple Choice	13	14	15	16	Total Score
60	10	10	10	10	100

Multiple Choice Questions

1. (5 points) Give the domain of

$$f(x) = \frac{\sqrt{3-x}}{x+2}.$$

A. $(-\infty, -2) \cup [3, +\infty)$
B. $[-2, +\infty)$
C. $(-\infty, -2) \cup (-2, 3]$
D. $[-2, 3]$
E. $(-\infty, 3]$

Need $3-x \geq 0$, $3 \geq x$

Need $x+2 \neq 0$, $x \neq -2$.

Thus. $(-\infty, -2) \cup (-2, 3]$.

2. (5 points) Let

$$f(x) = \frac{3x-2}{x+4}.$$

Find $f^{-1}(x)$ and its domain.

A. $f^{-1}(x) = \frac{x+4}{3x-2}$ with domain $(-\infty, 2/3) \cup (2/3, +\infty)$
B. $f^{-1}(x) = \frac{3x-2}{x+4}$ with domain $(-\infty, -4) \cup (-4, +\infty)$
C. $f^{-1}(x) = \frac{4x+2}{3-x}$ with domain $(-\infty, 3) \cup (3, +\infty)$
D. $f^{-1}(x) = \frac{4x+2}{3-x}$ with domain $(-\infty, -4) \cup (-4, +\infty)$
E. $f^{-1}(x) = \frac{4x-2}{3+x}$ with domain $(-\infty, -3) \cup (-3, +\infty)$

$y = \frac{3x-2}{x+4}$ Domain $x \neq -4$.

Reflect over line $y = x$

$x = \frac{3y-2}{y+4}$.

$(y+4)x = 3y-2$

$xy + 4x = 3y - 2$

$(x-3)y = -2-4x$

$y = \frac{-2-4x}{x-3}$.

$= -\frac{(2+4x)}{x-3}$

$= \frac{(2+4x)}{3-x}$

Domain $x \neq 3$.

3. (5 points) The position of an object is

$$s(t) = t^2 - 4t + 3$$

meters, where t is measured in seconds. What is its average velocity on $[1, 4]$?

A. -1 m/sec
B. -3 m/sec
C. 0 m/sec
D. 1 m/sec
E. 3 m/sec

ave velocity on $[1,4]$ $= \frac{s(4)-s(1)}{4-1}$

$= \frac{4^2-4(4)+3-(1^2-4+3)}{3}$

$= \frac{3-(0)}{3} = 1.$ meters/sec

4. (5 points) Let

$$f(x) = \begin{cases} 2x+1, & x < 3, \\ x^2 - 8, & x \geq 3. \end{cases}$$

Find $\lim_{x \to 3^-} f(x)$.

A. 3
B. The limit does not exist.
C. 1
D. -8
E. 7

$\lim_{x \to 3^-} f(x)$

$=$

$\lim_{x \to 3^-} (2x+1) = 2(3)+1$

$= 7.$

5. (5 points) Suppose

$$\lim_{x\to 2} f(x) = 3, \qquad \lim_{x\to 2} g(x) = -1.$$

Find

$$\lim_{x\to 2} \left[(x+1)^2 f(x) - 2xg(x)\right].$$

A. 35
B. 25
C. 13
D. 31
E. 39

$$\lim_{x\to 2}(x+1)^2 \cdot \lim_{x\to 2} f(x) - 2\cdot\lim_{x\to 2} x\cdot g(x)$$

$$= (2+1)^2 \cdot 3 - 2\cdot 2\cdot (-1)$$

$$= 9(3) - 4(-1)$$

$$= 27 + 4 = 31$$

6. (5 points) Use the Squeeze Theorem to find

$$\lim_{x\to 0} x^2 \cos\left(\frac{1}{x}\right).$$

A. The limit does not exist.
B. 0
C. 1
D. -1
E. The limit is infinite.

$$-1 \le \cos\left(\frac{1}{x}\right) \le +1$$

$$-x^2 \cdot 1 \le x^2\cos\left(\frac{1}{x}\right) \le x^2$$

Take limit

$$\lim_{x\to 0}(-x^2) \le \lim_{x\to 0} x^2\cos\left(\frac{1}{x}\right) \le \lim_{x\to 0} x^2$$

$$0 \qquad\qquad\qquad\qquad 0$$

By Sq. Thm, inside limit goes to 0.

7. (5 points) Give the largest set where the function $f(x) = \sqrt{2x-6}$ is continuous.

A. $[3, +\infty)$

B. $(-\infty, +\infty)$

C. $(-\infty, 3) \cup (3, +\infty)$

D. $[0, +\infty)$

E. $[0, 3]$

or

Need

$0 \leq 2x-6$

$6 \leq 2x$

$3 \leq x$

So $[3, +\infty)$

8. (5 points) Which statement correctly describes the behavior of

$$f(x) = \frac{5}{(x-2)^2}$$

near $x = 2$?

A. $\lim_{x \to 2^-} f(x) = +\infty$ and $\lim_{x \to 2^+} f(x) = -\infty$.

B. $\lim_{x \to 2} f(x) = 5$.

C. $\lim_{x \to 2^-} f(x) = +\infty$ and $\lim_{x \to 2^+} f(x) = +\infty$.

D. $\lim_{x \to 2} f(x) = 0$.

E. $\lim_{x \to 2^-} f(x) = -\infty$ and $\lim_{x \to 2^+} f(x) = +\infty$.

$\lim_{x \to 2^-}$ means take values closer & closer to 2 from left.

$\frac{5}{(\text{small } \#)^2} = \frac{5}{\text{smaller (+ positive)}} = +\infty.$

note negative inside square.

Similar $\lim_{x \to 2^+} = +\infty$ due to square in denom.

9. (5 points) Find

$$\lim_{x\to+\infty} \frac{4x^2-7x+1}{2x^2+5}.$$

A. The limit does not exist.

B. 0

C. 1/2

D. 2

E. $+\infty$

$$= \lim_{x\to+\infty} \frac{x^2\left(4-\frac{7}{x}+\frac{1}{x^2}\right)}{x^2\left(2+5/x^2\right)}$$

$$= \lim_{x\to+\infty} \frac{4-\frac{7}{x}+\frac{1}{x^2}}{2+5/x^2} = \frac{4-0+0}{2+0}$$

$$= 4/2 = 2$$

10. (5 points) If the tangent line to the graph of f at $x=4$ is $y=-2x+7$, give the value of the derivative $f'(4)$.

A. 0

B. -2

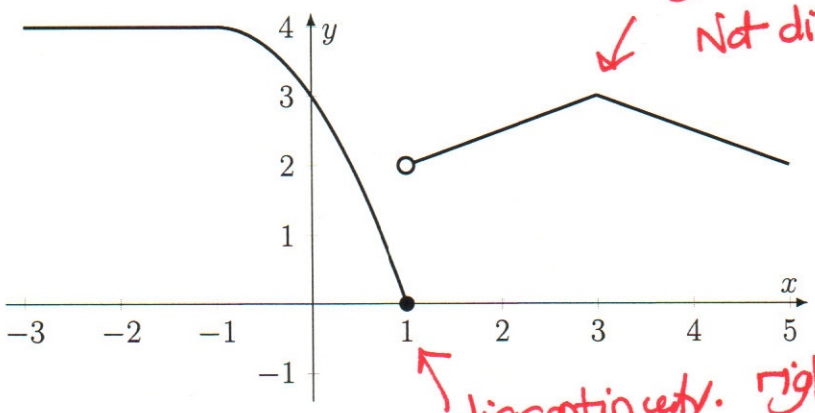
C. 4

D. 7

E. None of the above

$f'(4)$

The graph of f for Problems 11 and 12 is shown below.



corner, Not differentiable

discontinuity. right + left hand slopes

11. (5 points) The graph of f is shown at the top of this page. At which value of x does $f(x)$ fail to be continuous?

to cuve differ.

A. $x = 1$

B. $x = 0$

C. $x = 3$

D. $x = -1$

E. All of the above

12. (5 points) The graph of f is shown at the top of this page. At which values of x does $f'(x)$ fail to exist?

A. $x = 1$ and $x = 3$

B. $x = -1$ and $x = 3$

C. $x = -1$ and $x = 1$

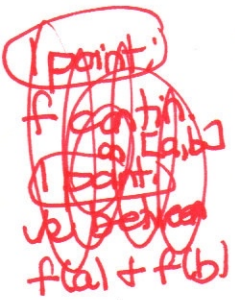
D. $x = 0$ and $x = 3$

E. $x = 0$ and $x = 1$

Free response questions, show all work, and clearly label your answers

13. (10 points) (a) State the Intermediate Value Theorem.

(b) Find an interval $[a, b]$ of length 1 (where a and b are integers) that contains a solution to the equation $x^3 + 2x = 15$. Use the Intermediate Value Theorem to prove that the interval you found contains a solution to this equation.



a. Let $f(x)$ be a function that is continuous on $[a,b]$. Let w be a value between $f(a)$ and $f(b)$. Then there is a value c in $[a,b]$ with $f(c) = w$.

1 point, 1 point, 1 point, 1 point

b. Let $f(x) = x^3 + 2x$. 1 pt

f is a polynomial, so it is continuous everywhere. 1 point

Let $w = 15$. 1 pt

Note $f(1) = 1^3 + 2(1) = 3$

$f(3) = 3^3 + 2(3) = 27 + 6 = 33$ 2 points

The value $w = 15$ is between $f(1) = 3$ and $f(3) = 33$.

Apply IVT to interval $[1,3]$, 2 points

there is a $c \in [1,3]$ with

$f(c) = 15$.

Free response questions, show all work, and clearly label your answers

14. (10 points) Evaluate each limit, or state that it does not exist. Show your steps clearly.

(a) $\lim_{x\to 3} \dfrac{x^2-9}{x-3}$

(b) $\lim_{x\to -4} \dfrac{x}{\sqrt{x^2}}$

(c) $\lim_{x\to +\infty} \dfrac{5x^2-1}{2x^2-3x+4}$

3pts total.

a. $\lim_{x\to 3} \dfrac{(x^2-9)}{x+3} = \lim_{x\to 3} \dfrac{(x+3)(x-3)}{x+3}$

$= \lim_{x\to 3} x-3 = 3-3 = 0$

2 pts to here. 1 pt.

3pts total.

b. $\lim_{x\to -4} \dfrac{x}{\sqrt{x^2}} = \dfrac{\lim_{x\to -4} x}{\lim_{x\to -4} \sqrt{x^2}}$

$= \dfrac{-4}{\sqrt{(-4)^2}} = \dfrac{-4}{\sqrt{16}} = \dfrac{-4}{4} = -1$

4pts total.

c. $\lim_{x\to +\infty} \dfrac{5x^2-1}{2x^2-3x+4} = \lim_{x\to +\infty} \dfrac{x^2\left(5-\frac{1}{x^2}\right)}{x^2\left(2-\frac{3}{x}+\frac{4}{x^2}\right)}$

$= \lim_{x\to +\infty} \dfrac{5-\frac{1}{x^2}}{2-\frac{3}{x}+\frac{4}{x^2}} = \dfrac{5-0}{2-0+0} = \dfrac{5}{2}$

2 pts for work. 2pts.

Free response questions, show all work, and clearly label your answers

15. (10 points) (a) State the definition of continuity of a function $f(x)$ at $x = c$.

(b) Consider

$$f(x) = \begin{cases} x^2 - 1, & x < 2, \\ a, & x = 2, \\ 2x - 1, & x > 2. \end{cases}$$

Determine all values of a for which f is continuous at $x = 2$, if any. Justify your answer.

3pts.

a. $f(x)$ is continuous at $x=c$ if $\lim_{x \to c} f(x) = f(c)$.

at $x=2$

b. To find limit, need right-hand & left-hand limits at $x=2$ to match.

3pts.

$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} x^2 - 1 = (2)^2 - 1 = 4 - 1 = 3$

$\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} 2x - 1 = 2(2) - 1 = 4 - 1 = 3$

So $\lim_{x \to 2^+} f(x) = \lim_{x \to 2^-} f(x) = 3$

implying $\lim_{x \to 2} f(x) = 3$

3pts { For continuity, need $f(2) = a = \lim_{x \to 2} f(x) = 3$ ↑ we found

1pt { Thus $a = 3$.

Free response questions, show all work, and clearly label your answers

16. (10 points) Let

$$f(x) = x^2 - 2x.$$

Use the limit definition of the derivative to find $f'(3)$. Your work should begin with writing $f'(3)$ in terms of a limit.

3pts.

$$f'(3) = \lim_{h\to 0} \frac{f(3+h) - f(3)}{h}$$

$$= \lim_{h\to 0} \frac{(3+h)^2 - 2(3+h) - (3^2 - 2(3))}{h}$$

$$= \lim_{h\to 0} \frac{9+6h+h^2 - 6 - 2h - (9-6)}{h}$$

3pts.

$$= \lim_{h\to 0} \frac{9 + 4h + h^2 - 6 - 3}{h}$$

$$= \lim_{h\to 0} \frac{4h + h^2}{h}$$

4pts.

$$= \lim_{h\to 0} 4 + h = 4 + 0 = 4$$