Further Lectures on Sets.

Ma 162 Spring 2010

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April 5, 2010

Product Sets.

- One of the important skills about sets is to be able to count the number of elements in a set. Quite often, a set is a table of information.
- Given sets A, B we can form the product set

$$A \times B \;=\; \{(a, b) \mid a \in A, b \in B\}.$$

- A little thought shows that if A, B are finite, then:

$$n(A \times B) = n(A) \cdot n(B).$$

- For example, we can have a class roll consisting of student names followed by letters giving the class grade. It will be convenient to consider the pair (Student Name, Grade) and call it an assigned grade.
- So, if S is the set of all students and G is the set $\{A, B, C, D, E, W\}$ then the assigned grades in a semester form the set $S \times G$.

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Multiple Products.

- To handle the whole grade book, we can let N be the set of integers between 0 and 100 and consider the product set

$$S \times N \times N \times N \times N \times G \text{ or } S \times N^4 \times G$$

whose members are 6-tuples consisting of student name followed by a sequence of four exam scores followed by the final grade.

- Naturally, we have an extended product formula

$$n(A_1 \times A_2 \times \cdots A_r) = n(A_1) \cdot n(A_2) \cdots n(A_r).$$

- We now illustrate how we can count the number of elements in various sets using this formula.
- For the same class of 180 students, we get

$$n(S \times N^4 \times G) = 180 \cdot 100^4 \cdot 6 = 108,000,000,000.$$

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Counting Specific Sets.

- The above number only gives the possible student records for a class of 180. The actual class records are, of course only 180, since there is exactly one per student.
  We will often need to identify and count specific subsets of the full product sets in this way.
- We cast a die 5 times and note the number on top, which would be a member of the set $S = \{1, 2, 3, 4, 5, 6\}$. If we record the 5 castings and record the top numbers in order, then we get members of the product set S^5 which has $6^5 = 7776$ elements.
- A telephone company assigns nine digit telephone numbers. How many different phones can it handle?
  We can imagine the telephone numbers as members of the set T^9 where $T = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$. Thus $n(T) = 10$, so $n(T^9) = 10^9 = 1,000,000,000$.

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A homework problem.

- Your favorite restaurant, Wally's Fine Dining, has a dinner menu with 6 appetizers, 11 entrees, and 8 desserts. A dinner at Wally's consists of 1 appetizer, 1 entree, and 1 dessert. What is the largest number of days could you eat dinner at Wally's without ever ordering the exact same meal?
- **Answer:** Imagine a choice card with three boxes marked appetizer, entree and dessert.
  They can be respectively filled with $6, 11, 8$ choices and by multiplication principle, the answer is the product: $(6)(11)(8) = 528$.

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Another Homework Problem.

- Manjula is extremely fashionable; so she can't stand wearing the same outfit twice to one job. She owns 6 shirts, 3 pairs of pants, and 9 pairs of shoes. If she works 270 days at her current job, how many more shirts must she get to have enough so that she will never have to wear the same outfit twice?
- **Answer:** Suppose she buys x new shirts. Then by multiplication principle, she would be good for $(6+x)(3)(9) = 162 + 27x$ days.
- We want this answer to become at least 270. Clearly $x = 4$ works! $(6+4)(3)(9) = 270$.

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More Telephone Numbers.

- Suppose we now require that the first digit of a telephone number cannot be 0 or 5, then we get a modified count $8 \cdot 10^8$ or $800,000,000$.
- This is typical. We often start restricting the elements of a product set and count the new subset.
- As another example, if besides the beginning 0 or 5, we also disallow all numbers with the same digit repeated 9-times, then we get to disallow these 10 numbers $000,000,000, 111,111,111, \cdots, 999,999,999$.
- Is the new answer $800,000,000 - 10$? A little thought will show that it is actually $800,000,000 - 8 = 799,999,992$.

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More Restricted Products: Permutations.

- A natural question about the telephone numbers can be the following. Suppose, we wish to count those telephone number which do not repeat any digits. We could try and calculate the numbers with some repeated digits and try to subtract them off.
- A little thought will show that this is not practical. There are too many ways to repeat some digit and it would be hard to keep track of double counting. A better strategy is the following: Imagine the permissible numbers as sequences of digits to be filled in. The very first digit can be any one of the 10 digits in T. The second can now be one of 9, since the first digit is now used up! The third has now 8 choices and continuing, the last (ninth) digit has only two choices left.
- Thus, the total count is $10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 = 3,628,800$.
- We end up with a much smaller answer!

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Permutations Continued.

- In general, many problems can be solved by this idea of filling in slots as we did above. The problem can be formulated as asking
- "in how many ways, can we seat n people in r chairs" or in a more neutral language, ' 'in how many ways can we arrange n objects in r positions"?
- The general answer is:

  $$P(n, r) = n \cdot (n-1) \cdot (n-2) \cdots (n-r+1).$$

  It is instructive to try various values of r and check this out!
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Circular permutations.

- Sometimes the problem is to seat a number of people in a circle rather than in a row of chairs.
- We can ask for the number of ways of arranging some r of n people at a circular table with r chairs.
- The simplest way to do this problem is to note that if the chairs are in a row, we know the answer $P(n, r)$. Now we move the chairs in a circle. We note that r different arrangements in a row can give the same arrangement in a circle since shifting everybody to the right in the circle does not give a new arrangement.
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Set Problems.

- Given that A, B and C are sets with $94, 67$ and 84 members respectively, answer the following.
- If $B \bigcap A$ has 45 members, then $B \bigcup A$ has ____ members.
  Answer: Use the formula

  $$n(B \bigcup A) = n(B) + n(A) - n(B \bigcap A) = 94 + 67 - 45 = 116.$$

- If it is further known that $C \bigcap A$ has 57 members, then $C \bigcup A$ has ____ members.
- **Answer:** By an identical calculation, we get:
  $94 + 84 - 57 = 121.$
- If, in addition $B - C$ has 40 members, then $C \bigcap B$ has ____ members.
- **Answer:** This time, we note that the sets $B - C$ and $C \bigcap B$ have no common elements and their union is B. So, we get

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- **Answer:** Using the three set formula:
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Combinations.

- Let us revisit our formula for permutations. Consider a problem of selecting a delegation of three students from a class of 25 students to go to a state convention.
- We could try to count the number of possible delegations thus. Set up three chairs and seat the students randomly in them one after the other.
- As we saw before, the possible number of such selections appears to be $25 \cdot 24 \cdot 23 = 13,800$. Do remember this as $P(25,3) = \frac{25!}{22!}$.
- But we need to think some more. Once the team is selected, the order does not matter. A little thought shows that the same 3-student team can appear in $3! = 6$ different ways in our selections, depending on the order of choosing.
- Since we are only interested in the team and not the order, the correct answer should be $13,800/6 = 2,300$.

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- We record the formula for choosing r element sets out of n objects as:

$$C(n,r) = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\cdots(n-r+1)}{r(r-1)\cdots(1)}.$$

- This is also described as the number of ways of choosing r objects out of n objects.
- We gave two forms of the formula. While the first is easy to remember, the second is easier to work with especially if $r < (n-r)$.
- Here are some observations and hints.
- $C(n,r) = C(n,n-r)$. Just check the first formula. Alternatively, think thus: Choosing r objects from n is the same as choosing $(n-r)$ for rejection!
- $C(n+1,r) = C(n,r) + c(n,r-1)$. Use simple algebraic manipulation.

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- $C(n,r) = C(n,n-r)$. Just check the first formula. Alternatively, think thus: Choosing r objects from n is the same as choosing $(n-r)$ for rejection!
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More Combinations.

- We record the formula for choosing r element sets out of n objects as:
$$C(n,r) = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\cdots(n-r+1)}{r(r-1)\cdots(1)}.$$
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Binomial Theorem.

- Binomial Theorem:

  $$(1+x)^n = 1 + C(n,1)x + C(n,2)x^2 + \cdots + C(n,r)x^r + \cdots + x^n.$$

- **Idea of the proof.** Think of $(1+x)^n$ as product of n terms $(1+x)$. To get a term x^r out of this product, we simply have to choose x from r of the terms and 1 from the remaining $(n-r)$ terms. Hence the term x^r must occur $C(n,r)$ times!
- Note that the above expression suggests why $C(n,0) = C(n,n) = 1$.
- Also, we must clearly have $C(n,r) = 0$ if $r > n$, since we cannot choose $r > n$ objects from among the n objects.
- $C(n,r)$ gives the number of ways of splitting an n-element set into two pieces - an r element set and an $(n-r)$ element set. We can ask for the number of ways of splitting into three sets of sizes $a, b, (n-a-b)$.

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- A similar argument can show that the answer is:

$$C(n,a,b) = \frac{n!}{(n-a-b)!a!b!}.$$

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For example:

$$(1+x+y)^3 = 1+3\,x+3\,y+3\,x^2+6\,xy+3\,y^2+x^3+3\,x^2y+3\,xy^2+y^3.$$

Verify these terms.

- **Food for thought!** Make suitable definitions and prove the full Multinomial Theorem.

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