

# Probability Exercises.

Ma 162 Spring 2010

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## Problem 1.

- **Conditional Probability:**

It is known that a student who does his online homework on a **regular basis** has a chance of **83 percent** to get a good grade (A or B); but the chance drops to **58 percent** if he **doesn't do the homework regularly**.

- John has been very busy with other courses and an evening job and figures that he has only a **69 percent** chance of **doing the homework regularly**.

What is his chance of **not getting a good grade** in the course?

- **Answer:** There are two important events under discussion. Let us name them.
- **E:** doing regular homework regularly.
- **F:** getting a good grade.

## Problem 1 Continued.

- There are two events  $E$  (regular homework) and  $F$  (good grade).
- We are given that  $P(F|E) = 0.83$  and  $P(F|E^C) = 0.58$ .  
We are also given that  $P(E) = 0.69$  and hence  $P(E^C) = 1 - 0.69 = 0.31$ .
- We get four possible paths to final outcome.
  - $E$  happens and then  $F$  happens. Thus  $E \cap F$  happens.
  - $E$  happens and  $F$  does not happen, i.e.  $E \cap F^C$  happens.
  - $E$  does not happen and  $F$  happens. Thus  $E^C \cap F$  happens.
  - Finally, neither  $E$  nor  $F$  happen, i.e.  $E^C \cap F^C$  happens.

## Problem 1 calculations.

- Note that:  
$$P(E \cap F) = P(E) \cdot P(F|E) = (0.69) \cdot (0.83) = 0.5727.$$
- We note that  $P(F^C|E) = 1 - P(F|E) = 0.17$ . So  
$$P(E \cap F^C) = P(E) \cdot P(F^C|E) = (0.69) \cdot 0.17 = 0.1173.$$
  
Note that this is also  $P(E) - P(E \cap F) = 0.69 - 0.5727$ .
- We can see that  
$$P(E^C \cap F) = P(E^C) \cdot P(F|E^C) = (0.31) \cdot (0.58) = 0.1798.$$
- Finally,  
$$P(E^C \cap F^C) = P(E^C) \cdot P(F^C|E^C) = (0.31) \cdot (0.42) = 0.1302.$$
- Thus, the probability of “not a good grade” ( $F^C$ ) is simply found by adding the cases with  $F^C$  in them. Thus the answer is  $0.1173 + 0.1302 = 0.2475$ , i.e. 24.75%.

## Problem 2.

- In an experiment, the probabilities of the events E, F and G are known to be 0.400, 0.300, and 0.320 respectively.
  - (i) If  $P(E \cap F) = 0$  then E and F are said to be independent/ mutually exclusive (choose one).
  - (ii) Suppose E and G are independent events. Then  $P(E \cup G) = ??$ .
- **Answer:** For (i), the correct choice is **mutually exclusive**. Recall that we declare two events E,G to be independent if  $P(E \cap G) = P(E)P(G)$ . So for part (ii) we can simply use our formula:  $P(E \cup G) = P(E) + P(G) - P(E \cap G) = P(E) + P(G) - P(E) \cdot P(G)$  and this evaluates to the answer:  
 $(0.4) + (0.32) - (0.4)(0.32) = 0.592$ .

## Problem 3.

- Bob and Sue are in a group of 10 people who are seated randomly around a table. What is the probability that Bob and Sue are seated next to each other?
- **Answer:** Here we set up a sample space of circular arrangements of 10 people around a table. By known formula, this number is  $n(S) = P(10, 10)/10 = 10!/10 = 9!$ . Next we count the number of sample points  $n(E)$  in our event, say  $E$ .

### Problem 3 continued.

- First, let Sue sit anywhere. Even though, this feels like 10 choices, it is **really just one choice due to circular arrangement!**.
- Next, let Bob sit next to her. This can be done in **2 ways** - on left or right.
- The rest of the people occupy the remaining 8 seats in  **$P(8, 8) = 8!$  ways**. Note that we no longer use circular arrangement, since our circle is now fixed with Bob and Sue at the head!
- By multiplication principle, the answer is:

$$\frac{n(E)}{n(S)} = \frac{(1)(2)(8!)}{9!} = 2/9.$$

### Problem 4.

- There are 40 different versions of an online homework. These are assigned randomly to the students. If a group of 3 classmates decide to work on the homework together, what is the probability that at least two of them receive the same version? **Hint:** It is easier to calculate the probability of this event not happening.
- **Answer:** The sample space consists of possible assignments of 3 of the 40 homeworks to our students, with repetition allowed! So, by multiplication principle  $n(S) = (40)(40)(40)$ .
- As suggested we consider the event  $E$  that our students get three different homeworks first. Clearly  $n(E) = P(40, 3) = (40)(39)(38)$ .
- So  $P(E) = \frac{(40)(39)(38)}{40^3} = \frac{1482}{1600}$ . Subtracting this from 1, we get the desired answer **0.074**.

## Problem 5.

- A raffle held by Mundane Products has collected 900 entries. They are giving away 60 TVs and 30 stereos. John bought 14 entries for the raffle.  
(i) What is the probability that John wins exactly one TV as well as exactly one stereo? (ii) John just heard that the TVs are already decided and he has won exactly one of the 60 TVs. What is the probability that John will win at least one stereo in the remaining drawing?
- **Answer:** The sample space has  $n(S) = C(900, 14)$  sample points of choices of 14 out of 900 tickets.
- Let  $E$  is the event of John winning 1 TV and 1 stereo and nothing more! Then he would have chosen one of the 60 TV tickets,  $(C(60, 1))$ , one of the 30 stereo tickets  $(C(30, 1))$  and remaining 12 loser tickets  $(C(810, 12))$ .

## Problem 5 continued.

- The desired probability is:

$$\frac{(60)(30)(810!)}{(12!)(798!)} \cdot \frac{(14!)(886!)}{900!}.$$

- This has to be calculated by simplification and either typing in the simplified fraction carefully or by careful evaluation. It comes out 0.1165.
- For part (ii), take out all the TV tickets and now the sample space has  $n(S) = C(840, 13)$ , since John still has 13 more tickets to choose.
- It would be easier to count the number of ways in which John does not win any stereos. That would be  $C(810, 13)$  when he chooses all loser tickets! This probability is  $\frac{C(810, 13)}{C(840, 13)}$  and the final answer is  $1 - \frac{C(810, 13)}{C(840, 13)} = 0.3789$ .

## Problem 6.

- Computer processors are shipped in lots of 70 from a factory. Before being shipped, 25 are randomly tested from each lot. If any of these 25 fail, the entire lot is not shipped. What is the probability that a lot containing exactly 4 bad processors gets shipped?
- **Answer:** The sample space has  $C(70, 25)$  elements obtained by counting the number of ways of selecting a sample of size 25.
- Our event consists of picking only the good processors and this can be done in  $C(66, 25)$  ways.
- So, the answer is  $\frac{C(66, 25)}{C(70, 25)}$ . This needs to be simplified and evaluates to 0.162.

## Problem 7.

- Due to economic downturn, a local factory is forced to lay off workers. The following table breaks down the lay offs.

	Being Laid Off	Not Being Laid Off	Total
Managers	95	215	310
Non-Managers	170	490	660
Total	265	705	970

- (i) What is the probability that an employee of the factory is being laid off given that s/he is a manager?
- (ii) Are the events “an employee selected at random is a manager” and “an employee selected at random is being laid off” independent?

**Note:** For this problem, we shall treat two probabilities as different if their difference is bigger than 0.05.

This is a relaxed test compared to 0.005 discussed in the lectures.

## problem 7 continued.

- For part (i), the calculation is simple. Of the 310 managers, 95 got laid off, so the probability is estimated as  $\frac{95}{310} = 0.306$ .
- For part (ii), we estimate probability of being a manager as  $\frac{310}{970}$  and the probability of being laid off as  $\frac{265}{970}$ . The product of these numbers is **0.0873**.  
The estimated probability of a manager being laid off from the given data is  $\frac{95}{970} = 0.0979$ .
- Since the difference between the two answers is less than 0.05, we estimate the probabilities as same, so the events are considered independent.

## Problem 8.

- What is the probability that at least 2 people in a random group of 6 people have a birthday on the same day of the week?
- **Answer:** There are 7 different weekdays and let us consider the probability of the complementary event of the six people having distinct birth weekdays.
- The probability is clearly  $\frac{P(7,6)}{7^6} = 0.0428$ .  
So, the probability of our event is  **$1 - 0.0428 = 0.9572$** .

## Problem 9.

- A box of 200 printer cartridges is used to stock the shelves. 20 of the cartridges are defective.
  - i) A customer buys 10 cartridges from the 200. What is the probability that all the ones he bought are good?
  - ii) A second customer buys 10 more cartridges from the same shelf. What is the probability that he got all good ones too, assuming the first customer got all good ones?
- **Answer:** As before, we set the sample space as number of choices of 10 cartridges out of 200 or  $n(S) = C(200, 10)$ .

## Problem 9 continued.

- For part (i), the event  $E$  consists of choosing of 10 cartridges which have no defective ones, so  $n(E) = C(180, 10)$ .  
So,  $P(E) = \frac{C(180,10)}{C(200,10)} = 0.3398$ .
- For part (ii), now we have only 190 cartridges with 20 defective. So, for the second customer, with similar calculation, we get the answer:  $\frac{C(170,10)}{C(190,10)} = 0.3195$ .



## Problem 10.

- Suppose you are dealt a hand of 4 cards at random from a deck of 32 cards consisting of 4 aces, 4 2s, 4 3s, and so on up to 4 8s.
- i) What is the probability that your hand will have at least 3 hearts?
- ii) What is the probability that your hand will have at least 3 cards in the same suit?
- **Note:**  
The cards were taken from an ordinary deck of cards, so there is 1 ace of each suit, one 2 of each suit, etc.

## Problem 10 continued.

- The sample space consists of choosing 4 cards from 32, so  $n(S) = C(32, 4) = 35960$ .
- For part (i), we want to count the number of ways of choosing at least 3 heart cards.  
This happens in exactly two ways. We may have 3 hearts and one non heart, or all four hearts.
- Since there are 8 heart cards, we can choose three in  $C(8, 3)$  ways and the fourth card can be chosen in  $C(24, 1) = 24$  ways. By multiplication principle, the count is  $C(8, 3)(24) = 1344$ .
- There are  $C(8, 4) = 70$  ways of choosing all four hearts.  
Thus the required probability is  $\frac{1344+70}{35960} = 0.0393$ .
- Part (ii) is now easy. We simply repeat the argument for each suit and note that the answers add up since the events are mutually exclusive. So the answer is:  
 $(4)(0.0393) = 0.1573$ .

## Problem 11.

- In an experiment, two events  $E$  and  $F$  are known to have probabilities 0.500 and 0.650 respectively, and it is suspected that the events are independent. However the probability of the event  $E \cap F$  cannot be determined theoretically.

So the experiment is run 200 times and the event  $E \cup F$  occurred 199 times. Based on this data, you would tend to think that the events  $E$  and  $F$  are/are not (choose one) independent, because the product of 0.500 and 0.650 is too large / too small / close to what it should be (choose one).

- **Answer.** We estimate  $P(E \cap F)$  by the formula  $P(E) + P(F) - P(E \cup F) = 0.5 + 0.65 - 199/200 = 0.155$ .
- Independence would require  $(0.5)(0.65) = 0.325$  to be close to this estimated probability 0.155. Clearly, the product is too large and the event are not independent!