Understanding the Simplex algorithm.

Ma 162 Spring 2010

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March 1, 2010

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- Moreover, in the current course we assume that $B \ge 0$. This insures that the choice X = 0 satisfies all the inequalities, i.e. is a feasible solution.

Problems can be analyzed without this assumption, but we won't try to solve them not in this course.

• Here is an example (4.1 Example 3): Take

$$A = \begin{bmatrix} 2 & 1 & 2 \\ 2 & 4 & 1 \\ 1 & 2 & 3 \end{bmatrix}, B = \begin{bmatrix} 14 \\ 26 \\ 28 \end{bmatrix} \text{ and } C = \begin{bmatrix} 2 & 2 & 1 \end{bmatrix}.$$

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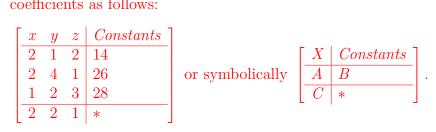
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1	2	3	28		C	*
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Minimize YB subject to $YA \ge C$ and $Y \ge 0$.

- The reason to write Y as a row rather than a column is somewhat technical and would be clarified below.
- The amazing theorem called the "duality theorem" states that any solution of the original maximization problem by the Simplex Algorithm produces a solution to its dual minimization problem by simply reading the final tableau. We describe this next.

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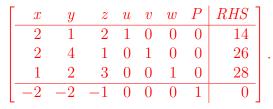
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$$\begin{bmatrix} x & y & z & u & v & w & P & RHS \\ \hline 1 & 0 & 7/6 & 2/3 & -1/6 & 0 & 0 & 5 \\ 0 & 1 & -1/3 & -1/3 & 1/3 & 0 & 0 & 4 \\ 0 & 0 & 5/2 & 0 & -1/2 & 1 & 0 & 15 \\ \hline 0 & 0 & 2/3 & 2/3 & 1/3 & 0 & 1 & 18 \end{bmatrix}$$

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• Inspection of the final tableau says that the final basis is x, y, w, P and hence the final basic solution is:

- The Voodoo Principle also tells us the actual row transformations that we performed. This information is read from the 4 × 4 matrix under the variables *u*, *v*, *w*, *P*.
- We know that the row transformations can be performed by multiplying the original 4×8 matrix by some 4×4 matrix on the left. We see that this transformation matrix must be

$$\left[\begin{array}{c|c} M & 0 \\ \hline Y & 1 \end{array} \right] =$$

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- By looking at the the entries at the foot of the x, y, z columns, we deduce that

$$\begin{bmatrix} 0 & 0 & 2/3 \end{bmatrix} = -C + YA$$

since the original entries were $-C = \begin{bmatrix} -2 & -2 & -1 \end{bmatrix}$ and we added YA to it. Thus $YA \ge C$.

• By looking at the last entry in the bottom row, we know that it was 0 and we have added *YB* to it.

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• Recall that the two Linear Programming Problems (LPP) are:

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Maximize: P = CX s.t. X \ge 0, AX \le B
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and

Minimize: Q = YB s.t. $Y \ge 0, YA \ge C$.

- Recall that by a feasible solution to either problem we mean a solution which satisfies all the inequalities, but may not give the maximum or minimum.
- If X_0 , Y_0 are feasible solutions to the two problems respectively, then we see that $Y_0B \ge Y_0AX_0 \ge CX_0$. Thus the function value $Q_0 = Y_0B$ is always bigger than or equal to the function value $P_0 = CX_0$.

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• Thus, if X_0 and Y_0 are feasible solutions to the maximization and its dual minimization problems respectively and if

$$P_0 = CX_0 = Y_0B = Q_0$$

- Thus for our dual problems $X_0 = (5, 4, 0)$ and $Y_0 = (2/3, 1/3, 0)$ are the respective solutions of the maximization and the minimization problems with a common function value 18.
- Warning! Note that the Y values are read at the foot of the original slack variables, but they are not the values of the basic solution for the slack variables corresponding to the maximization problem.

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- Recall that we always assume all our variables to be non negative in this course. In the following discussion, we are only discussing the remaining inequalities; we shall call them essential inequalities.
- If our essential inequalities are of ≤ type with non negative RHS, then we write it as a maximization problem and solve with the Simplex algorithm. (If we happen to be minimizing a function, we can always maximize its negative instead!)
- If our essential inequalities are of ≥ type with non negative RHS, then we write it as a minimization problem and solve its dual with the Simplex algorithm. Then read off the solution under the slack columns as shown above. (If we happen to be maximizing a function, we can always minimize its negative instead!)
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11 / 12

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