

This worksheet gives hints on how to do some homework problems similar to those on D1, D2. The problems themselves are from older homeworks.

commands

>

>

Homework D1

Q.1.

If you enter the lottery by choosing 4 different numbers from a set of 47 numbers, how many ways are there to choose your numbers?

Answer: Use the $C(n,r)$ formula.

$$C(47,4) = \frac{47(46)(45)(44)}{4!} = \frac{178365}{1} = 178365$$

What if you have to also choose one more number from a separate set of 51 numbers (like the "Powerball" lottery)? How many ways are there to pick all of your numbers if you include choosing this last number?

Answer: Multiply the above by the extra choice 51, so

> **numbcomb(47,4);**

178365

> **%*51;**

9096615

Q.2.

A baseball team has 15 members, 4 of whom always play and bat at the top of the batting order. For the game Sunday, the starting lineup of nine players will be filled out from the remaining members and a batting order will be set.

How many possible batting orders can be made?

Answer: Here the order is important, so we need the $P(n,r)$ formulas.

The top 4 players are arranged in the top four slots by $P(4,4) = 4! / (4-4)! = 4! / 0! = 24$.

The bottom 5 slots are filled from the remaining 11 players, so $P(11,5) = 11! / (11-5)! = 11! / 6!$

55440
= .

The final answer is the product of the two numbers $24 * 55440$.

> **numbperm(4,4);**

24

> **numbperm(11,5);**

55440

> **%*%*%;**

1330560

Q.3.

In an experiment, a six-sided die is rolled once. Describe the event "the number rolled is odd or the number rolled is greater than or equal to 4" by checking sample points included in that event.

Answer: The condition is satisfied by the numbers 1,3,5,4,5,6, so we check these.

Q.4.

This Sunday, Ann wants to have open house at 3 homes of the 13 homes she has listed currently. In how many possible ways can she do this?

Answer: We have to "choose" 3 houses from 13 so $C(13,3) = 286$.

> **numbcomb(13,3);**

286

Q.5.

In a local boxcar race, only 8 contestants can enter. This year, though, 11 kids entered their names. The judges have decided to draw names from a hat to decide who can race this year. The starting position of contestants will be decided by when their name is drawn (1st name = 1st position and so on).

In how many different ways can the judges assign all the starting positions?

Answer: The order matters! We have 8 slots and 11 kids, so we want:

$$P(11,8) = \frac{11!}{3!} = 6652800.$$

> **numbperm(11,8);**

6652800

Q.6.

An experiment consists of studying the hair color of all members of families with one child. Below is a listing of the sample space for this experiment with each letter corresponding to the hair color of a member of the family (Father, Mother, Child - in that order). There are two possible hair colors considered: L=light, D=dark.

Describe the event "the child does not have dark hair" by selecting all the sample points included in it.

Answer: The condition corresponds to the last marker being "not D", i.e. being "L". So we check all choices which end in "L".

Q.7.

Bill's new CD player has a 5-disc changer. If he owns 20 CDs, in how many ways can Bill choose 5 CDs to listen to?

Answer: ok

After picking 5 CDs, in how many ways can he order them in the 5 slots of the changer?

Answer: First he "chooses" 5 from 20, so $C(20,5) = \frac{15504}{5!}$. Next, these are "arranged in 5 slots, so $P(5,5) = 5! = 120$.

> **numbcomb(20,5);**

15504

> **numbperm(5,5);**

120

Q.8.

A dinner party has 8 couples. The hostess wishes to seat all the guests around a circular table alternating males and females. How many different seating arrangements are there? Assume that there is no special seat around the table.

Answer: Let us first seat 8 ladies in alternate seats. This is a circular arrangement, so $P(8,8)/8 = 8!/8 = 7!$.

Next, we seat the men in the gaps. Even though these are in a circle, a rotation around the circle will give a new seating, since the men will have different ladies on their sides. So, the number of ways of seating the men is $P(8,8)=8!$.

203212800.

The answer is $7! \cdot 8! =$

> **(numbperm(8,8)/8)*numbperm(8,8);**

203212800

Q.9.

A teacher is trying to pick 7 students to represent her class at an upcoming school competition. If there are 14 boys and 11 girls in her class, in how many ways can the teacher pick 4 boys and 3 girls to attend the event?

Answer: The boys and girls are "picked" or "chosen" separately and the results multiplied.

165165.

So $P(14,4) \cdot P(11,3) =$

> **numbcomb(14,4)*numbcomb(11,3);**

165165

Q.10.

A florist has access to 12 types of flowers and 9 types of foliage. He makes bouquets using 3 types of flowers and 2 types of foliage.

How many different bouquets can he make?

Answer: This is another "choosing" problem, since the order of the chosen types does not matter.

Hence: $P(12,3)*P(9,2) = 7920$.

> **numbcomb(12,3)*numbcomb(9,2);**

7920

Q.11.

How many ways can the letters of the word "kentucky" be arranged?

Answer: First we act as if all letters are distinct, so we get $P(8,8)$. Then this is divided by various factorials. If some letter is repeated r times, it is divided by $r!$. This is done for each repeated letter.

Thus, here $8!/2! = 20160$.

As another example, for "hawaii" we shall get $6!/(2!2!)$ since "a" and "i" are repeated twice.

For "illinois", we get $8!/(3!2!)$.

> **numbperm(8,8)/numbperm(2,2);**

20160

Q.12

How many ways can 5 knights out of a group of 7 knights be chosen and seated at a round table?

Note: two seatings are the same if the people seated are the same and the counterclockwise order of seating is the same for both seatings.

Answer: This is a circular arrangement of 7 knights in 5 slots, so $P(7,5)/5 = 504$.

> **numbperm(7,5)/5;**

504

>

>

Homework D2

Q.1.

For extra security, a high school searches 5 random lockers each school day. If there are 680 students with lockers, what is the probability a particular student's locker gets searched at least once in a 30 day period? Hint:

First determine the probability of getting picked in a given day and then use it!

Answer: Imagine a sample space of 680 lockers with the simple events of a given locker

being searched. Since all are equally likely, each has a probability of $\frac{1}{680}$ and since 5

are searched on a given day $\frac{5}{680}$ is the probability of some specific locker being picked on a given day.

a little thought will say that it is easier to figure out the probability of not being picked for 30 days and subtract it from 1 for the desired answer.

$$1 - \frac{5}{680} = \frac{675}{680}$$

Thus, the probability of not being picked on a given day is

$$\left(\frac{675}{680}\right)^2$$

Now, the probability of not being picked two days in a row shall be and more generally the probability of not being picked for n days in a row shall be

$$\left(\frac{675}{680}\right)^n$$

$$\left(\frac{675}{680}\right)^{30}$$

This says that the probability of **not being picked** in 30 days is . Hence our

$$1 - \left(\frac{675}{680}\right)^{30}$$

final answer is . This can be entered as a formula (recommended method) or evaluated and then substituted!

> (675.0/680)^30;1-(675.0/680)^30;

0.8013934775

0.1986065225

Q.2. In an experiment, the probability of the event E is known to be .5. Also the probability of the event $F \cup E$ is 1.

the event F is .8, and the probability of

i) Compute the probability of these events:

$$P(F \cap E)$$

$$P(E \setminus F)$$

ii) Suppose the experiment is run twice in succession. What is the probability of the event E occurring at least once?

Answer: (i) We use the formula $P(E \cup F) = P(E) + P(F) - P(E \cap F)$ to get

$$1 = .5 + .8 - P(E \cap F)$$

$$.5 + .8 - 1 = .3$$

So the first answer is

$$P(E \setminus F) = P(E) - P(E \cap F)$$

For the second answer, we note that . So we get:

$$P(E \setminus F) = .5 - .3 = .2$$

or

(ii) We note that the probability of E not occurring is $1 - .5 = .5$ So, probability of E not occurring twice in a row is $.5^2 = .25$. Thus, our desired event (which is the complement of this) has probability $1 - .25 = .75$.

Q.3. Two not necessarily different times are chosen from twelve possible times, 1 o'clock thru 12 o'clock. What is the probability the times are within 3 and a half hours of each other?

Answer: Here the correct sample space needs to be chosen. The intended space has $\frac{12(11)}{2} = 66$ points consisting of pairs of different timings between 1 to 12 (or $C(12,2) = 66$) and 12 choices of using the same time twice. We assume that these are all equally likely, each with probability $\frac{1}{78}$.

For our desired event, suppose that the first time is 12 and the second time is same or later - so 12, 1, 2 or 3. This gives 4 choices. Since this can be done for each of the 12 first times, we have $4(12) = 48$ or 48 sample points. Thus the total probability is $\frac{48}{78} = \frac{8}{13}$.

Comment: It is possible to argue with conviction that the event of picking a pair of different times should be considered to be twice as likely as picking a pair of identical times. It would lead to a different probability calculation since in that case the probability

of a pair of different times will come out $\frac{2}{144}$ and that of two identical times will be $\frac{1}{144}$. Then our event will have probability $12 \left(\frac{1}{144} + \frac{3(2)}{144} \right) = \frac{7}{12}$. But that is not the intended answer here!

Q.4. A raffle sells 71 tickets numbered 1 through 71 and one ticket is drawn at random. What is the probability that the number drawn is a multiple of 3 or 7?

$$\frac{1}{71}$$

Answer: The sample space has 71 points each with probability $\frac{1}{71}$. For our event, we can count the number of sample points thus.

The multiples of 3 can be 3,6,9,...,69 or 23 in all. the multiples of 7 are 7,14,...,70 or 10 in all. But if we think that $23+10$ is the answer, then there is an error. We are counting 21, 42,63 twice since they are both multiples of 3 and 7. So the correct count is $23+10-3 = 30$.

$$\frac{30}{71}$$

The probability is $\frac{30}{71}$.

>

Q.5.

A blue die and a white die are rolled. What is the probability that the blue die will land with a 2 in the uppermost position and the white die will land with a 5 in the uppermost position? (Select the correct box.)

$$6(6) = 36$$

Answer: Our sample space has 36 sample points consisting of the pairs of

$$\frac{1}{36}$$

numbers on top of the two dice in order. each sample point has probability $\frac{1}{36}$. Our

$$\frac{1}{36} = .278\text{e-}1$$

event is a simple event (2,5) so the answer is $\frac{1}{36}$.

What is the probability that the blue die will land with a 2 in the uppermost position or the white die will land with a 5 in the uppermost position (or both)? (Select the correct box.)

Answer: In this case the event has many point. With the blue die with 2, the red die can give 6 cases and similarly with the red die with 5 you get 6 cases. These two counts both

$$6 + 6 - 1 = 11$$

count the case of 2-blue and 5-red. So the correct count is 11. The answer is

$$\frac{11}{36} = .306$$

.

> evalf(1/36,3);

0.0278

> **evalf(11/36,3);**

0.306

Q.6.

A survey of home-owners found that 44% own less than 2 cars, 58% own less than 3 cars, and 10% own 4 or more cars. What is the probability that a randomly chosen home-owner owns 2 or 3 cars?

Answer: Let us make our sample space consisting of four simple events.

A= The event "Owns 0 or 1 cars".

B=The event "Owns 2 cars".

C=The event "Owns 3 cars".

D=The event "Owns 4 or more cars".

These are all disjoint cases and hence can be thought of as simple events.

We are given that $P(A) = \frac{44}{100}$, $P(A \cup B) = P(A) + P(B) = \frac{58}{100}$, $P(D) = \frac{10}{100}$.
 $\frac{58 - 44}{100} = \frac{14}{100}$

It follows from the first two equations that $P(B) = \frac{14}{100}$ or $\frac{14}{100}$.

Also since all the four events should add up to probability 1, we get

$$\frac{44}{100} + \frac{14}{100} + P(C) + \frac{10}{100} = 1 \quad 1 - \frac{68}{100} = \frac{32}{100}$$

or $P(C) = \frac{14 + 32}{100} = \frac{46}{100}$.

the desired answer is $P(B) + P(C) = \frac{14}{100} + \frac{46}{100} = \frac{60}{100} = 0.6$.

What is the probability that a randomly chosen home-owner owns exactly 2 cars?

$$\frac{14}{100}$$

Answer: As already found: $\frac{14}{100}$.

Q.7.

A particular cell phone provider conducts an internal study of call failures. The study finds that 4% of calls fail to reach the network from the caller's phone, 5% fail to be received by the second party's phone, and 4% are the result of a failure in the network. What is the overall probability that a random cell phone call will fail to connect?

Note that from the wording, you need to assume that only one of the faults occurs at a time. The reason is that once a fault occurs, the call fails and the other faults don't have a chance to occur!

If the faults could occur simultaneously, then the answer may not be known without further information.

Answer: As explained, the three types of failures are mutually exclusive events, so their probabilities simply add up. The answer is $4+5+4 = 13\%$ or 0.13 .

Q.8.

In a game of blackjack, each player is dealt two cards from a standard 52-card deck. If the first card is 3 of spades, what is the probability that the second card is 8 of hearts?

Answer: The denominations of the cards themselves are irrelevant.

After the first card is dealt, there are 51 cards left and we are simply asking for the

$$\frac{1}{51}$$

probability of a specific card, so the answer is .

Q.9.

Mr. Phillips, an optician, estimates that the probability that a customer coming into his store will purchase one or more pairs of glasses is .40, the probability that he will purchase one or more pairs of contact lenses is .25, and the probability that he will purchase both contacts and glasses is .15. Hence, Mr. Phillips concludes that the probability that a customer coming into his store will purchase a pair of glasses or a pair of contact lenses is .50.

Is this true or false?

Answer: Let us name the events:

A = the event of purchasing glasses

B=the event of purchasing contact lenses.

Then we are given $P(A) = 0.40$, $P(B)=0.25$ and $P(\overset{A \cap B}{A \cup B}) = 0.15$.

We need to determine $P(\overset{A \cup B}{A \cap B})$.

By the formula, it is $0.40+0.25 - 0.15 =0.50$. So, his conclusion is correct!

Q.10.

Let E and F be events associated with the same sample space. Describe the event that "E or F occurs" using the set notation for unions, intersections, and compliments.

Choose between **A:** $E \cup F$ **B:** E intersect F **C:** $E \cap F^C$ **D:** $F \cap E^C$

Answer: Recall that "or" means union, "and" means intersection. The notation in C means E minus F and the one in D means F minus E.

So, we choose A as the answer.

Q.11

Suppose you are dealt a hand of 3 cards at random from a deck of 32 cards consisting of 4 aces, 4 2s, 4 3s, and so on up to 4 8s.

i) What is the probability that your hand will have at least 2 4s?

Answer: We can set up the sample space as the choices of three cards from 32, i.e. there

$$\frac{32!}{3! 29!} = \frac{32 (31) (30)}{3!}$$

are C(32,3) sample points. This number is . No need to evaluate this, yet!

We need to count the number of sample points in our event. There are two possibilities.

1. We can pick all three "4's" and the number of choices shall be $C(4,3) = C(4,1)=4$.

2. We can pick two "4's" and one "non 4".

This number shall be $C(4,2)*C(28,1) = 6*28$. The number 28 comes from counting the cards which are not "4's" and the calculation is $32-4$, since there are four "4's".

Now we are ready to calculate the probability as $(4+6*28)/C(32,3) = 172/4960$

$$.3467741935e-1$$

If we need a decimal number, it would be . But a fraction is a good enough answer.

ii) What is the probability that your hand will have no two cards of the same numerical value (that is, at most one ace, at most one 2, ect.?)

Answer: As we know there are eight different denominations(numerical values) and four cards of each denomination. Thus to count our event, we choose three of the eight denominations in C(8,3) ways and then choose one card out of four from each of them .This second choice can be done in $4*4*4 = 64$ ways.

$$\frac{8 (7) (6) (64)}{6}$$

Thus the total count is $C(8,3)*64 = 3584$.

$$\frac{3584}{4960}$$

The probability is then .

```
> numbcomb(32,3);
```

```
4960
```

```
> (4+28*6);
```

```
172
```

```
> evalf(172/4960);
```

```
0.03467741935
```

```
> numbcomb(8,3)*4^3;
```

```
3584
```

```
>
```