THE BAKHSHALI MANUSCRIPT

Svami Satya Prakash Sarasvati
&
Dr. Usha Jyotishmati

THE
BAKHSHALI MANUSCRIPT

AN ANCIENT TREATISE OF INDIAN ARITHME

EDITED BY

Svami Satya Prakash Sarasvati

and

Dr. Usha Jyotishmati, M. Sc., D. Phil.

Dr. Ratna Kumari Svadhyaya Sansthan

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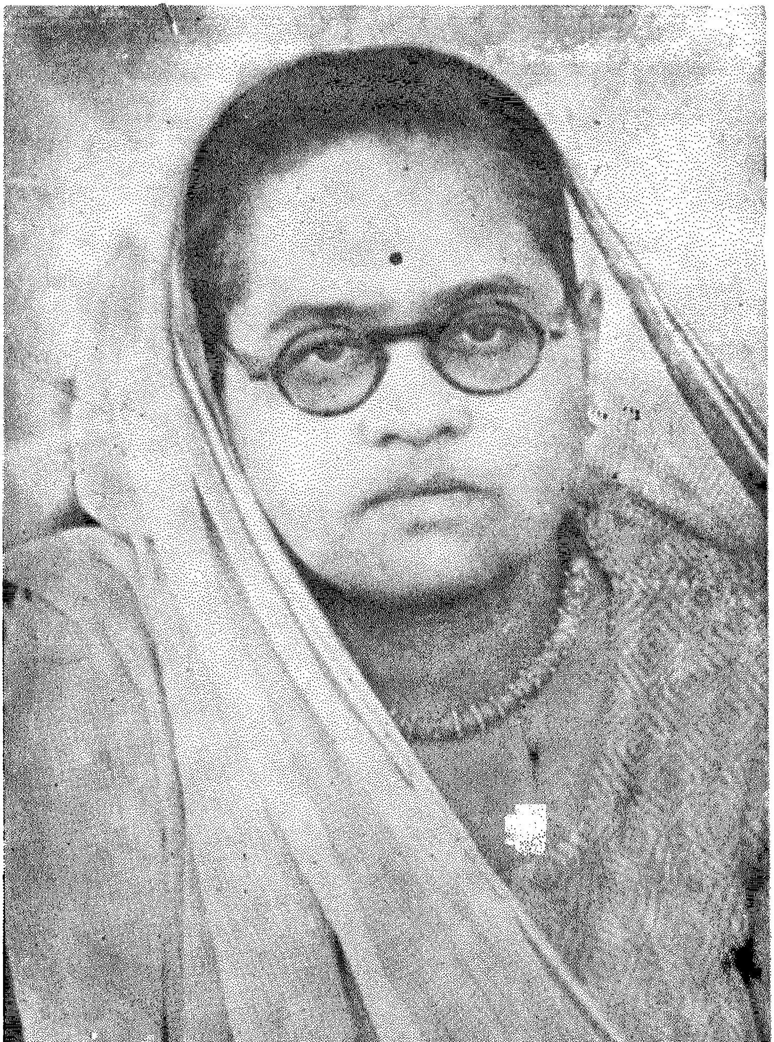


Dr. Ratna Kumari

Born 20-3-1912 Died 2-12-1964

PREFACE

Dr. Ratna Kumari, M. A., D. Phil. was deeply interested in education, higher research and scholarship, and when she died in 1964, the Director of the Research Institute of Ancient Scientific Studies, New Delhi, graciously agreed to publish in her commemoration a Series to be known as the "*Dr. Ratna Kumari Publication Series*", and under this arrangement, the five volumes published were: *Satapatha Brahmanam*. Vol. I, II and III (1967, 1969, 1970); *Baudhayana Sulba Sutram* (1968) and the *Apastamba Sulba Sutram* (1968). It is to be regretted that in 1971, Pandit Ram Swarup Sharma, the Director of the Institute died and shortly afterwards, the activities of the Institute came to a close. In 1971, from an endowment created by the relations of late Dr. Ratna Kumari, *Dr. Ratna Kumari Svadhyaya Sansthana*, a research organization for promotion of higher studies amongst ladies, was established at Allahabad, with Sri Anand Prakash, the younger son of Dr. Ratna Kumari as the first President. Svami Satya Prakash (formerly, Prof. Dr. Satya Prakash) has authorised Dr. Ratna Kumari Svadhyaya Sansthana to publish several of his works, particularly, all of them which were published by the Research Institute of Ancient Scientific Studies, West Patel Nagar, New Delhi, and has assigned the right of publication of these works to the Sansthana.

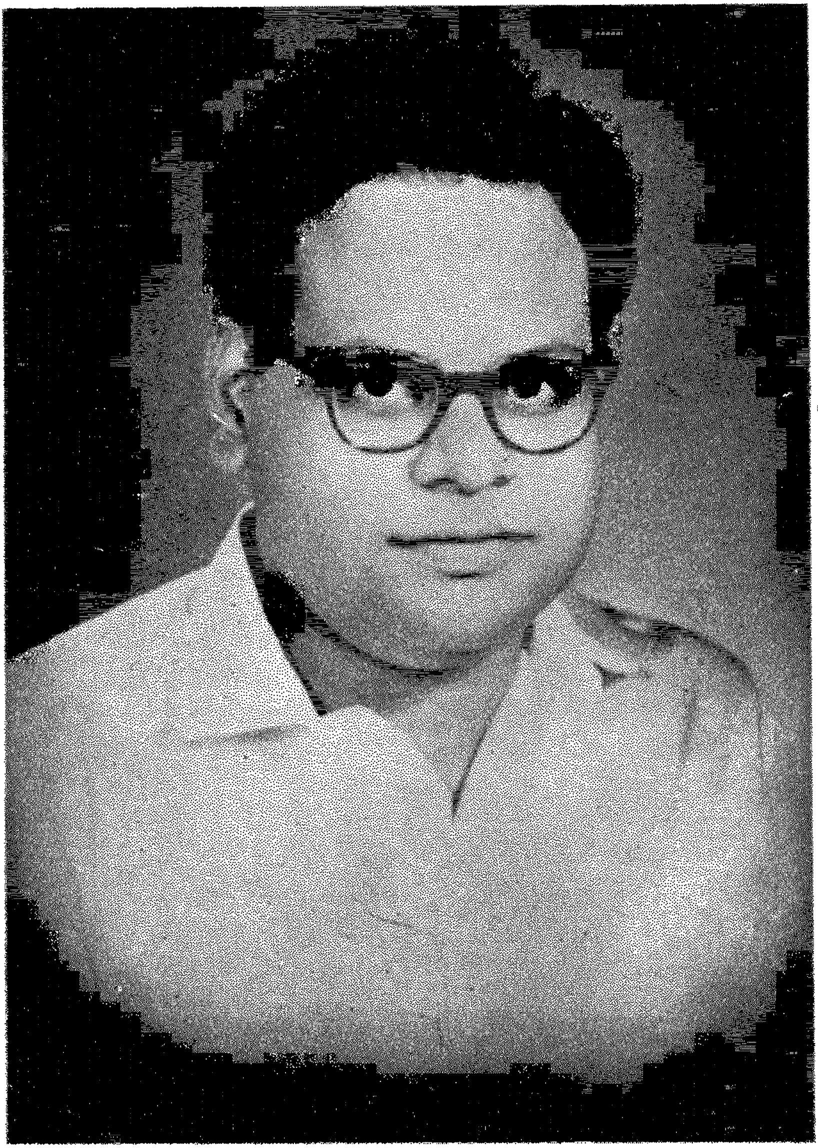
We are pleased to offer to the public for the first time, the classified text of the *Bakhshali Manuscript* in Devanagari script. The manuscript was literally dug out of a mound at Bakhshali on the north-west frontier of India in 1881, and Dr. Hoernle carefully examined the manuscript and published an English translation of a few of the leaves in 1888. This being the earliest manuscript on Indian Arithmetic, it aroused considerable interest in the West. The Archaeological Survey of India published the manuscript in Parts I, II and III, as edited by G. R. Kaye. In 1902, Dr. Hoernle presented the manuscript to the Bodleian Library. This distinguished work of Kaye is now not available in market. Dr. Bibhuti Bhusan Datta of Calcutta University also made detailed study of this manuscript.

Dr. Usha Jyotishmati, a distinguished member of the Sansthana, took considerable pains in editing the text, and we are obliged to Svami Satya Prakash Sarasvati for his supervision and critical introduction. We are also obliged to Sri Jagdish Prasad Misra, B A., LL. B., for his dedicated services to the Sansthana, particularly in the publication programmes.

We regret to announce the premature death of our first President Sri Anand Prakash on December 13, 1976. He was a distinguished graduate of the University of Allahabad, and took his graduation in Mechanical Engineering from the University of Glasgow, and since then he had been working in industry. To cherish his memory, we have the privilege of dedicating this little volume.

The Sansthana
Allahabad
Ramanavami
April 5, 1979.

Prabha Grover
M Sc., D. Phil.
Director



Anand Prakash

Born 1-4-1938 Died 13-12-1976

To Our President

ANAND PRAKASH

who died at

an early age of 38.

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INTRODUCTION

Chajakputra—The Scribe :

While we pay homage to the unknown author of a text which came to be known as the *Bakhshali Manuscript*, we should express our gratitude to another person of unknown name who scribed the manuscript from a text quite lost to us, and not only that, since this scribe was himself a mathematician of no less repute, being known as a prince amongst calculators, he added illustrations and often added something to the running commentary of this Text. Though we have no independent source to corroborate, it is definite that a Text existed as eariy as the second or third century A. D. if not earlier; this Text was traditionally used by the lovers of calculations (teachers and taughts) in some parts of the country (specially the north-western parts of the Aryavartta), and as it was handed down to us generation after generation, new worked and unworked problems were added to the existing material. At the end of the manuscript (or somewhere on Folio 50, *recto*) we find a colophon mentioning that the work was scribed by a certain Brahmana, a prince of calculators the son of Chajaka (छजक). Thus we know the name of the father of the scribe only. I would like to call the scribe of the present manuscript himself as Chajaka-Putra (छजक पुत्र), the son of Chajaka. He himself is not an author of the Text; he merely copies it out from some other text already current. Even if the date of copying this text were so late as the tenth or eleventh century and even if some illustrations included in the manuscript were not of a very early origin, the Bakhshali Text has its great importance, this being one of the earliest Texts in history available to us on the science of calculations.

How the Text was Found ?

In 1881, a mathematical work written on birchbark was found at Bakhshali near Mardan on the north-west frontier of India. This manuscript was supposed to be of great age and its discovery aroused considerable interest. Part of it was examined by Dr. Hoernle, who published a short account of it in 1883[1], and a fuller account in 1886[2], which together with the translation of a few of the leaves was republished in 1888[3]. Dr. Hoernle had intended, in due course, to publish a complete edition of the Text, but was unable to do so. The work was later on printed and published in 1927, under the title "*The Bakhshali Manuscript*", Part I and II, by the Government of India with photographic fascimiles and transliteration of the Text together with a very comprehensive introduction by G. R. Kaye[4]. This was followed by the publication of the *Bakhshali Manuscript*, Part III, in 1933 as the *Text Rearranged* shortly after Kaye's death. Dr. Bibhutibhushan Datta, a distinguished worker in the field of Indian mathematics, also published a critical review on the *Bakhshali Manuscript*[5].

Bakhshali (or Bakhshalai, as it was written in the official maps) is a village of the Yusufzai subdivision of the district of Peshawar of the North-Western Frontier of India (now in Pakistan). It is situated on, or near, the river Mukham, which eventually joins the Kabul river near Nowshers, some twenty miles further south. Six miles W. N. W. of Bakhshali is Jamalgarhi, twelve miles to the West of Takht-i-Bhai and twenty miles W. S. W. is Charsada, famous for their Indo-Greek art treasures.

Bakhshali is about 150 miles from Kabul, 160 from Srinagar, 50 from Peshawar, 350 from Balkh and 70 from Taxila. It is in the Trans-Indus country and in ancient times was within Persian boundaries—in the Arachosian satrapy of the Achaemenid kings. It is within that part of the country to which the name Gandhara

1. *Indian Antiquary*, XII (1883), pp. 89—90.
2. Verhandtungen des VII Internationalen Orientalisten Congresses, Arische Section, pp. 127 et. seq.
3. *Indian Antiquary*, XVII (1888), pp. 38—48, 275—279.
4. The *Bakhshali Manuscript*—A Study in Medieval Mathematics, Part I and II, Calcutta, 1927. Kaye made two previous communications on the subject matter of the Bakhshali work: (i) Notes on Indian Mathematics—Arithmetical Notation (*J. Asiat. Soc. Beng.*, III, (1907) and (ii) The *Bakhshali Manuscript, ibid.* VIII, 1912.
5. Datta : *Bulletin of the Calcutta Mathematical Society*, 1929, XXI, p. 1, entitled *the Bakhshali Manuscript*. Also references in the *History of Hindu Mathematics*, By Datta and A. N. Singh, 1935 and 1962 editions.

has been given, and was subject to those western influences which are so bountifully illustrated in the so-called Gandhara art.

The authentic record of the discovery of the manuscript appears to be contained in the following letter dated the 5th of July, 1881[1], from the Assistant Commissioner at Mardan.

"In reply to your No. 1306, dated 20th ultimo, and its enclosures, I have the honour to inform you that the remains of the papyrus MS. referred to were brought to me by the Inspector of police, Mian An-Wan-Udin. The finder, a tenant of the latter, said he had found the manuscript while digging in a ruined stone enclosure on one of the mounds near Bakhshali[2]. These mounds lie on the west side of the Mardan and Bakhshali roads and are evidently the remains of a former village. Close to the same spot the man found a triangular-shaped 'diwa', a soap-stone pencil, and a large *lota* of baked clay with a perforated bottom. I had a further search made but nothing else was found.

"According to the finder's statement the greater part of the manuscript had been destroyed in taking it up from the place where it lay between stones. The remains when brought to me were like dry tinder, and there may be about fifty pages left some of which would be certainly legible to any one who knew the characters. The letters on some of the pages are very clear and look like some kind of *Prakrita* (प्राकृत), but it is most difficult to separate the pages without injuring them. I had intended to forward the manuscript to the Lahore Museum in the hope that it might be sent on thence to some scholar, but I was unable to have a proper tin box made for it before I left Mardan. I will see to this on my return from leave. The papyrus will require very tender manipulation. The result will be interesting if it enables us to judge the age of the ruins where the manuscript was found[3]."

1. Apparently the manuscript was found in May 1881.

2. General Cunnigham in a private letter to Dr. Hoernle, dated Simla, 5th June, 1882, says : "Bakhshali is 4 miles north of Shahbazgarhi. It is a mound with the village on the top of it The birch-bark manuscript was found in a field near a well without trace of any building near the spot, which is outside the mound village......".

3. Kaye has, however, expressed doubts regarding the authenticity of this account, for, he thinks, it was written apparently from memory, a month or so after the discovery of the manuscript.

In the meantime notices of the discovery had found their way into the Indian newspapers. Professor Buhler, who had read of the discovery in the "Bombay Gazette,[1] communicated the announcement to Professor Weber, who brought it to the notice of the Fifth International Congress of Orientalists then assembled in Berlin. In Professor Buhler's letter to Prof. Weber it was stated that the manuscript had been found "carefully enclosed in a stone chamber" and it was thought that the newly discovered manuscript might prove to be "one of the Tripitakas which Kaniska ordered to be deposited in Stupas".

Kaye, however, says that there is nothing whatever in the record of the find to justify Buhler's statement, which seems to have originated in a rather strange interpretation of the words "while digging in a stone enclosure" that occur in the letter quoted above, and which are themselves of doubtful reliability.

The manuscript was subsequently sent to the Lieutenent-Governor of the Punjab, who, on the advice of General Cunningham, directed it to be transmitted to Dr. Hoernle, then head of the Calcutta Madrasa, for examination and publication. In 1882, Dr. Hoernle gave a short description of the manuscript before the Asiatic Society of Bengal, and this description was published in the *Indian Antiquary* of 1883. At the Seventh Oriental Conference, held at Vienna in 1886, he gave a fuller account which was published in the proceedings of the Conference, and also with some additions, in the

1. This accont appears in the Bombay Gazette of Wednesdav, August 13th, 1881 and is as follows :

"The remains of a very ancient papyrus manuscript have been found near Bakhshali, in the Mardan Tahsil, Peshawar District. On the west side of the Mardan and Bakhshali road are some mounds, believed to be the remains of a former village, though nothing is known with any certainty regarding them, and it was while digging in a ruined stone enclosure on one of these mounds the discovery was made. A triangular-shaped stone '*Diwa*', and a soap-stone pencil and a large *lotah* of baked clay, with a perforated bottom, were found at the same place. Much of the manuscript was destroyed by the ignorant finder in taking it up from the spot where it lay between the stones; and the remains are described as being like dry tinder, in some of the pages However, the character, which somewhat resembles 'prakrita'(प्राकृत), is clear. and it is hoped it may be deciphered when it reaches Lahore, whither we understand it is shortly to be sent.".

Indian Antiquary of 1888. In 1902 Dr. Hoernle presented the manuscript to the Bodleian Library.

The Manuscript

The manuscript consists of some 70 leaves of birchbark, but some of these are mere scraps. The largest leaf measures about 5.75 by 3.5 inches or 14.5 by 8.9 centimetres. The leaves, which are numbered according to the Bodleian Library arrangement from 1 to 70, may be classified according to their size and condition as follows :—

In fair condition but broken at the edges—size, not less than 5 by 3 inches (13 by 8 cms.)

1, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 20, 22, 23, 24, 32, 33, 34, 36, 37, 43, 44, 47, 49, 59, 60, 61, 62, 63 · Total 35.

Rather more damaged but otherwise in fair condition not less than 4¾ by 2 inches (12 by 5 cms.)

2, 25, 26, 42, 45, 46, 48, 50, 51, 52, 55, 56, 57, 58, 64, 69 · Total 16.

Much damaged

21, 31, 35, 41, 5 , 66, 68—Total 7.

Scraps.

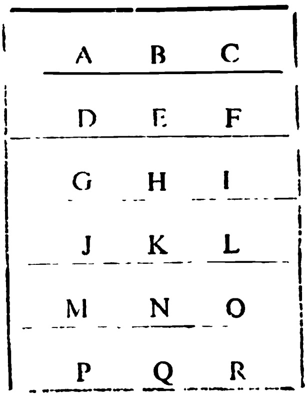
27, 28, 29, 30, 35, 38, 39, 40, 54, 68, 70—Total 11.

One folio (19) is entirely blank.

Certain folios consist of two leaves stuck together, namely 7, 31 and 65, and possibly others. It would not be difficult to separate these double leaves without damaging the manuscript.

The leaves are now mounted between sheets of mica and placed within an album. The mica sheets are about 7.4 by 4.6 inches and are fixed together by strips of gummed paper at the edges leaving a clear area of 6¼ by 3¾ inches.

Possibly the original strip of birch-bark from which the leaves of Bakhshali manuscript were taken was roughly of the shape of the annexed diagram and was cut up into the oblongs indicated. If A, B, C, etc., represent the upper layer, and A′, B′, C′, etc. the lower layer, then, according to the evidence of the leaves themselves, they were arranged for purposes of writing upon in the order A, A′; B, B′; C, C′; etc., or A, A′; D, D′; etc.



Regarding the format of the Bakhshali manuscript (ratio 1.7) as a criterion of age, Kaye says that he could come to no positive conclusion from the evidence before him. Dr. Hoernle, however, writes as follows :—

"It is noteworthy that the two oldest (Indian paper) manuscript known to us point to their having been made in imitation of such a birch bark prototype as the Bakhshali manuscript."

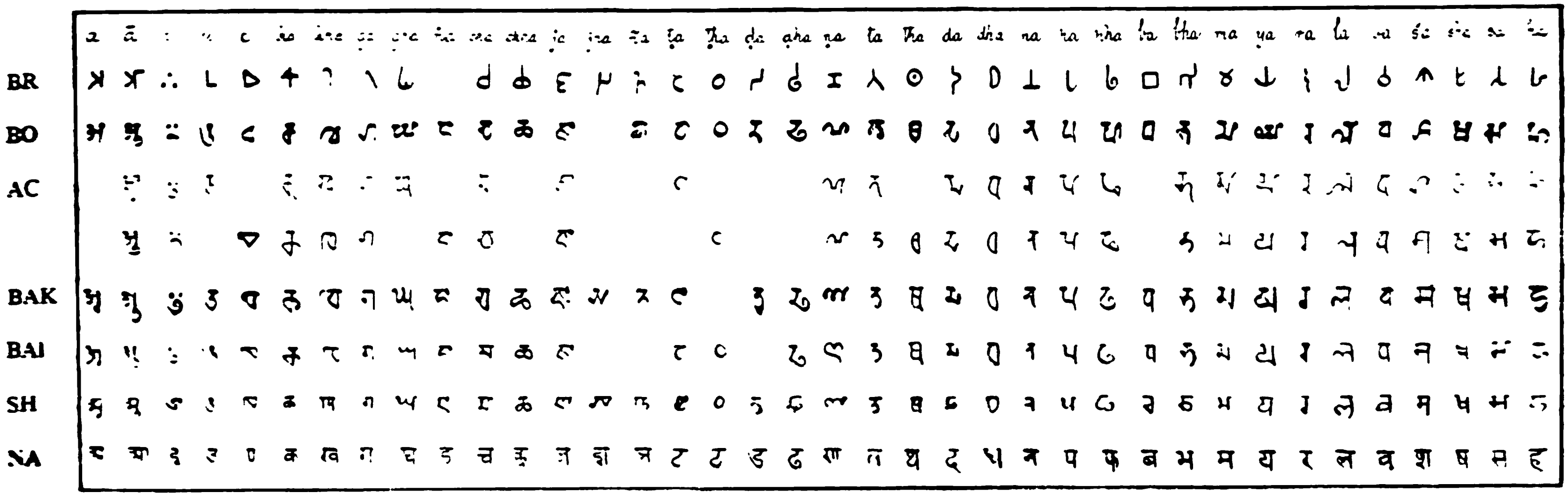
Kaye, however, does not accept this argument, for it would be quite as reasonable to conclude that the Bakhshali format was determined by the paper manuscript formats, and that it is of later date than the introduction into India of paper as a writing material; and this, according to Kaye, would place the Bakhshali manuscript about the twelfth century of our era at the earliest.

The Script

The Bakhshali text is written in the Sarada (शारदा) script, which flourished on the north-west borders of India from about the Ninth Century until within recent times. Its distribution in space is fairly definitely limited to a comparatively small area lying between longitudes 72 and 78 east of Greenwich and latitudes 32 and 36 north Dr. Vogel distinguishes between Sarada proper, of which the latest examples are of the early Thirteenth Century, and modern Sarada.

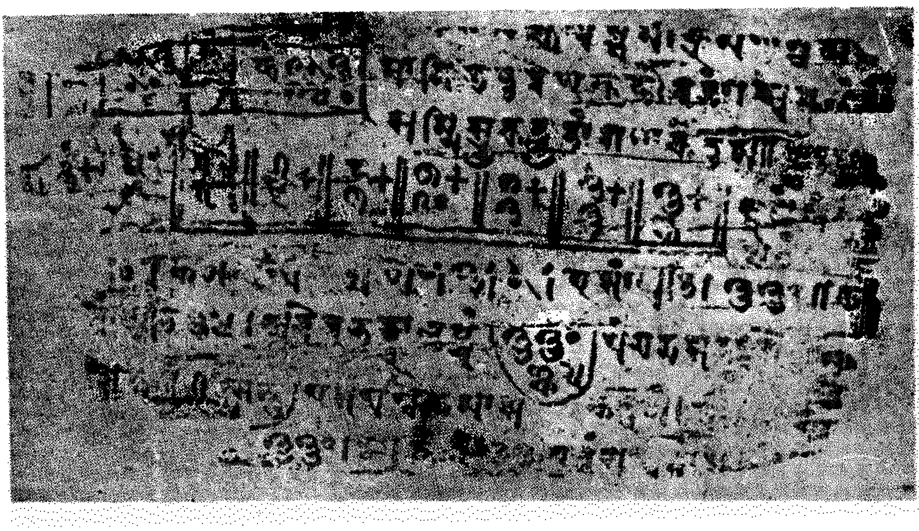
The writing of the Bakhshali manuscript is of the earlier period and is generally very good writing indeed. It was written by at least

Old Indian Scripts



BR = Brahmi ; BO = Bower Ms. ; AC = Acute-angled ; SA = Sarahan ; BAK = Bakhshali Ms. ; BAI = Baijnath ;
SH = Sharada (Modern) ; NA = Nagari.

A page of the Bakhshali Manuscript
(17 verso)



two scribes. Kaye gives a detailed account of it, and has classified the writing styles also (*a* and *β* as he denotes them). Style *a* is divided into four sub-sections which possibly belong to the work of four separate scribes, although it is not easy to point out any fundamental differences between these styles. Folio 65 possibly exhibits the writings of two separate scribes on the two sides, which do not belong to the same original leaf.

The style *β* is distinguished by its boldness by ' tails'' or flourishes (including very long *virama* marks), the methods of writing mediale, *ai* and *o*, the looped six, etc , etc.

A particular section, which Kaye calls as M, has no example of *ai* and contains practically all the examples of "*o*". Sections L (a_4) and G (a_3) show marked differences in the methods of writing medial *e*, while section F is, in this matter, very much like section L. Section C (a_2) has no example of either the *JIHVAMULIYA* (जिह्वामू-लीय) or *UPADHMANIYA*, (उपध्मानीय), and so on; but it must be borne in mind that these statistics are only of value in the mass.

The Language

The language of the text may be described as an irregular Sanskrit. Nearly all the words used are Sanskrit, and the rules of Sanskrit grammar and prosody are followed with some laxity. The peculiarities of spelling, *sandhi*, grammar, etc. that occur in the text are exceedingly common in the inscriptions of the Eleventh and Twelfth Centuries found in the north-west of India. Dr. Hoernle, however, implies that the language is much older. He states that the text "is written in the socalled Gatha dialect, or in that literary form the North-Western Prakrita which which preceded the employment, in secular composition, of the classical Sanskrit". He also states that this dialect "appears to have been in general use, in North-Western India, for literary purposes, till about the end of the Third Century A. D".

Order and Arrangement of Folios

The Bodleian Library order of the folios, which was definitely fixed by circumstances not within the control of the editor, has necessarily been followed by Kaye in his edition, in the arrangement of the facsimiles and the first transliteration of the taxt (Part I and Part II). The whole material was rearranged in the posthumous publication : *The Bakhshali Manuscript*, Part III.

The leaves were disarranged to some extent before they reached Dr. Hoernle, but unfortunately he did not leave any proper record of the order in which the leaves reached him.

Dr. Hoernle attempted to arrange the leaves on the basis of the numbered *sutras*; but the numbered *sutras* were two few and too unevenly distributed to serve this purpose, and in some ways they were even misleading. The chief criterion of order is, of course, the nature of the contents of the leaves, but Kaye made an attempt to utilise all available criteria as help towards the solution of the problem of order. Had all the leaves been extant, even if fragments, the problem could have been completely solved; but some leaves are completely missing and many fragments have disappeared altogether, so that the problem could be only partially soluble Sometimes, even the knots in the birch-bark were of assistance, and besides this natural aid there was the accidental one of the effects of the method of storage. Possibly for some hundreds of years the bundle of leaves was subject to a certain amount of pressure, and was exposed particularly at the edges, to chemical and other disintegrating actions. Some of the leaves stuck together, the edges of all became frayed and certain leaves became so frail as to break up into scraps when handled. On the principle that contiguous leaves would be affected approximately to the same extent, we might, if no disintegrating effects had taken place since the find, rebuild in layers the original bundle. But we know that further disintegration has taken place; however, similarity in size and shape and mechanical makings were of distinct help in rearranging the leaves.

Contents of the Manuscript

The portions of the manuscript that have been preserved are wholly concerned with mathematics. Dr. Hoernle described the work in 1888 in the following words :—

"The beginning and end of the manuscript being lost, both the name of the work and its author are unknown. The subject of the work, however, is arithmetic. It contains a great variety of problems relating to daily life. The following are examples :-

(i) 'In a carriage, instead of 10 horses, there are yoked 5, the distance traversed by the former was one hundred; how much will the other horses be able to accomplish'[1] ?

(ii) The following is more complicated :- 'A certain person travels 5 *yojanas* on the first day, and 3 more on each succeeding day ; another who travels 7 *yojanas* on each day, has a start of 5 days ; in what time will they meet ?'[2]

(iii) The following is still more complicated :—'Of 3 merchants, the first possesses 7 horses, the second 9 ponies, the third 10 camels; each of them gives away 3 animals to be equally distributed among themselves. The result is that the value of their respective properties become equal : how much was the value of each animal ?'[3]

"The method prescribed in the rules for the solution of these problems is extremely mechanical and reduces the labour of thinking to a minimum".

The following is a summary list of the contents of the work as far as its present state allows of such analysis :—

	Section
Problems involving systems of linear equations	A
Indeterminate equations of the second degree	A and K
Arithmetical progressions	B and C
Quadratic equations	C
Approximate evaluations of square-roots	C
Complex series	F
Problems of the type $x\,(1-a_1)\,(1-a_2)\ldots(1-a_n)=P$	G

1. नियोरथोश्वैर्दशभिर्युज्यते हय पञ्चकम् । गन्तव्यं योजनशतम्
...किमुद्भवेत् । | ह १० हय लग्नरथस्य ५ / १ १ | ।गन्तव्ययोजन १०० / १ |

—Folio 8, verso.

2. आदि पञ्चम उत्तरं त्रीणि नरो योजन गम्यते । द्वितीय प्रतिदिनंस्सप्तगतस्य दिन पञ्चकम् । केन कालेन समतां कथ्यतां गणकोत्तम ।

—Folio 6, recto

3. ...द्वितीयस्य हयान् नवः ऊष्ट्रा दश तृतीयस्य...प्रदत्त च परस्परं पृथग् धनं तु वणिजां मूल्यं वा प्राणिनां पृथक् यदि......वक्तुं ततो मेच्छिछग्धि संशयः ।

—Folio 3, verso.

The computation of the fineness of gold	H
Problems on income and expenditure, and profit and loss	L, D and E
Miscellaneous problems	M
Mensuration	

Such is a very rough outline of the work as it now stands. Perhaps the most interesting sections are C, A and M; and of these C is the most complete and was evidently treated as of cosiderable importance. Section A is also of special interest as it contains examples which may be described as of the *epanthcm* type. Section M is of interest principally on account of the methods of expressing the numerous measures involved and also because of its literary and social references.

Generalized Arithmetic and Algebra

Although the work is arithmetical in form it would be misleading to describe it as a simple arithmetical text-book No algebraical symbolism is employed, but the solutions are often given in such a general form as to imply the complete general solution, i. e., the solutions, though arithmetical in form, are really generalized arithmetic, or algebra. First of all a particular rule is given, which is intended to apply to the particular set of examples that follows. These rules are often expressed in language that would be impossible to interpret without the light thrown upon them by the solutions. The examples are themselves sometimes trivial, but the solutions, often expressed with what at first glance appears to be meticulous care, often redeem the examples from their apparent triviality. Proofs or verifications are often given with some elaboration and on occasions are multiplied.

Kâye has said that the work may be divided roughly into algebraic, arithmetical and geometrical sections, knowing of course that the boundaries of these sections are not clear, but according to him it would be more correct to classify the problems as (a) academic, (b) commercial, aud (c) miscellaneous.

Judging by the manuscript as it now stands, the problems involving geometric notions were comparatively very few, and we can only guess at the meanings of the remaining fragments dealing with this branch of mathematics.

One pleasing feature is the small space occupied by commercial problems. There is only one problem on interest, and a rather unobtrusive section containing problems on profit and loss.

Those problems classed as academic are concerned with particular mathematical notions that in early mediaeval times had a traditional value and interest, such as the *epanthema*, the *regula virginum*, certain indeterminate equations of the second degree, and certain sets of linear equations.

The miscellaneous problems include examples where the chief interest is rather in the illustrative material than in the mathematical notions involved; e. g., there are problems concerned with the abduction of Sita by Ravana, the prowess of Haihaya, the constitution of an army, the Sun's chariot, the daily journey of the planet Saturn, gifts to Siva, etc. etc.

Such, or similar features, are, however, common to many mediaeval mathematical works. The Bakhshali manucript is, however, almost unique in at least two respects of some mathematical importance: (i) The first of these is the employment of a special sign in the form of a cross—exactly like our *plus* sign but placed after the quantity it effects—to indicate a negative quantity. (ii) The second special characteristic consists of the set of methods for indicating the change-ratios of certain measures.

Text is of Indian form

Whether of a purely Indian origin or not, Kaye says that the work is Indian in form. It is written in a sort of Sanskrit and generally conforms to the Indian text-book fashion but there are certain apparent omissions. Perhaps the most noteworthy feature of the classical Hindu texts is their treatment of indeterminate equations of the first degree, while their greatest achievement is the full solution of the so-called Pellian equation. A great part of the texts of Brahmagupta, Mahavira and Bhaskara are devoted to one or both of these topics, but there is no evidence of either in what remains of the Bakhshali text; and this apparent omission is the more noticeable, inasmuch as there is evidence of considerable skill in the treatment of systems of linear equations and certain indeterminates of the second degree. Another omission to note is of a different character altogether. Every early Hindu work of this kind has a section relating to the "shadow of a gnomon", but in our text there is no evidence of such a section. We must not, however, as Kaye says, "pay too much attention to these apparent omissions. The possibility of entire sections of the manuscript being destroyed is not great, but negative evidence and a mutilated manuscript do not carry us very far.

Is the Work Homogeneous ?

On the first examination of the manuscript it would be noticed that the writing was not uniform, and that, in particular, certain leaves differentiated themselves from the rest by a bolder and,on the whole, a better style of writing; and Kaye has distinguished this set of leaves as the "M" section. This early differentiation was a most useful one for it marked not only a difference in style of writing but also one of matter. Indeed this "M" section proved to have so many peculiarities that the idea that it was possibly a separate work could not be ignored. But the rest of the manuscript is by no means uniform in style of writing or anything else, and Kaye is not convinced that the "M" section is the work of a separate author, although, he rather suspects that it is. It is, however, pretty certain that it was the work of a separate scribe; but, as there are slight indications that the other portions of the manuscript were dictated, this does not affect the question of authorship conclusively.

The peculiarities of the "M" section, although they may not prove heterogeneity of workmanship, call for some special mention and are here summarized.

1. The Script :

(a) The writing is bolder and, on the whole, more uniform than that of the rest of manuscript.

(b) Flourishes or extensions of the bottom endstrokes are common. These flourishes occur particularly at the end of ligatures, but also in the cases of the numeral figures "5", "7" and "9", and even in the case of the stop bars, and occasionally they even occur at the ends of the frame-works of the "cells" (e. g., see fol. 47, etc.).

(c) The numerical symbols of section "M" have been shown in a table by Kaye [not reproduced here, Table IV (7) line 1], where the looped "6" should be noted. This is useful but not an infallible criterion,

(d) The following table relating to the formation of medial vowels a, i, and o taken from Part II.

	Medial ai		Medial o'		
	ai	ai	o'	o'	o'
'M' section	0%	100%	32%	32%	36%
Whole, manuscript	24%	76%	75%	15%	10%

Here the indication appears to be very definite indeed, but it must be borne in mind that such criteria only apply in the mass, that the total number of ai examples is only 19, and so on. [1]

II. It is curious that all the mythological and semi-historical references (see Sections 47-48) occur in the "M" section. Indeed this section is peculiarly Hindu in contrast with the remainder of the manuscript.

III. The mathematical contents of the "M" section may be described as miscellaneous problems, which are generally solved by "rule of three", but a special feature is the occurrence of numerous "measures" and a special method of exhibiting their change-ratios. But, of course, these points in no way indicate work—rather otherwise for if section "M" were an entirely different work we might expect some duplication, and there is none here.

IV. The method of exposition is somewhat different. The example is followed by a statement and the answer is then given, generally, without any detailed working, and generally there is no "proof" or verification. There are, however, exceptions.

V. There are differences in language. Certain technical terms that are extremely common in the rest of the manuscript do not occur' e g , *pratyaya* and *yuta*.

An Indigenous Hindu Treatise

Dr. Hoernle believes that arithmetic and algebra developed in India entirely on indigenous lines. According to him "in the Bakhshali manuscript, there has been preserved to us a fragment of an early Buddhist or Jain work on arithmetic (perhaps a portion of larger work on astronomy), which may have been one of the sources

(1) The symbols used here are merely mnemonic,

from which the later Indian astronomers took their arithmetical information." Again according to Hoernle, the arithmetic in India developed without any Greek influence, "Of the Jains, it is well-known, says Hoernle, "that they possess astronomical books of a very ancient type, showing no tracees of westeren or Greek influence" (Here is a probable reference to the *Suryaprajnapti,*). In India arithmetic and algebra are usually treated as portions of works on astronomy. In any case, it is impossible that the Jains should not have possessed their own treatises on arithmetic, when they possessed such on astronomy. The early Buddhists too are known to have been proficients in mathematics. The prevalence of Buddhism in North-Western India, in the early centuries of our era, is a well-known fact."

Kaye, however definitely asserts that there is not the slightest evidence in the manuscript itself of its being connected either, with the Jains or Buddhists. It is Hindu (Saivite); the author was a Brahmana (fol. 50); to Siva, he attributes the gift of calculation to the human race (fol.50); offerings to Siva are mentioned on more than one occasion (fols. 34. 44); references are made to certain incidents recorded and persons named in the Hindu epics (fol. 32 etc.) and there is *not a single reference* that could be construed as indicating any connection with Buddhism and Jainism.

Various Opinions about its Age.

The original Bakhshali work has been assigned various dates by several scholars. Hoernle says : "I am disposed to believe that the composition of the former (the Bakhshali work) must be referred to the earliest centuries of our era, and that it may date from the third or fourth century A. D.[1]" This estimation about the age of the original Bakhshali work has been accepted as fair by eminent orientalists like Buhler[2] and historians of mathematics like Cantore[3] and Cajori[4]. Thibaut has followed Hoernle in accepting the date of the present manuscript to be lying between 700 and 900 A. D.[5] But Kaye would refer the work to a period about the twelfth century. "The script, the language. the contents of the work", says he, "as far as they can give any chronological evidence,

(1) *Indian Antiquary*, XVII, p. 36. (2) *Indian Palaeography, p. 82.*

(3) M. Cantor, *Geschichte der Math*, I. P. 598.

(4) F. Cajori, *"History of Mathematics"*, 2nd ed. Boston, 1922 p. 85.

(5) G. Thibaut, *Astronomie, astrologie und mathematik, p. 75.*

all point to about this period, and there is no evidence whatever incompatible with it"[1], Bibhutibhusan Datta agrees with Hoernle in that the work was written towards the beginning of the Christian era, and he has discussed this issue in his paper, "*The Bakhshali Manuscript*" published in the Bulletin of the Calcutta Mathematical Society, 1929, XXI, p. 1. We shall present his arguments here.

Bakhshali Mathematics Older Than the Present Manuscript

Hoernle thinks that the mathematical treatise contained in the Bakhshali manuscript is considerably older than the present manuscript itself. "Quite distinct from the question of the age of the manuscript", says he, "is that of the work contained in it. There is every reason to believe that the Bakhshali arithmetic is of a very earlier date than the manuscript in which it has come down to us."[2], This conclusion has been disputed and rejected by Kaye who thinks it to be based on unsatisfactory grounds. He then adds, "of course it will be impossible to say definitely that the manuscript is the original and only copy of the work but we shall be able to show that there is no good reason for estimating the age of the work as different from the age of the manuscript to any considerable degree.."[3] Kaye has adversely criticised the linguistic and palaeographic evidence of Hoernle. Datta, however, thinks that Kaye's arguments, if proved sound and sufficient, will establish at the most that the present manuscript was written about the twelfth century, as is contended by him.[4] Hoernle himself considers it to be not much older, belonging probably to a period about the ninth century of the Christian era.[5] Most of the other reasons of Kaye against Hoernle's view, based on certain internal evidence, such as (i) the the general use of the decimal place value notation, (ii) the occurence of the approximate square-root rule and (iii) the employment of the *regula falsi*, on imperfect knowledge of the scope and development

(1) *Bakh. Ms.*, § *135.*

(2) *Indian Antiquary*, XVII, p. 36

(3) *Bakh. Ms.* § *122* (4) *Bakh. Ms.* § *135*

(5) *Indian Antiquary*, XVII, p. 36.

§. In support of this opinion, Kaye states : "There is evidence that Ms. is not a copy at all. It is not the work of a single scribe : there are cross references to leaves of the manuscript; there is a case of wrongly numbering a *Sutra* and the mistake is noted in another hand-writing" (p. 74 fn). The facts noted in the latter part of this statement cannot possibly support what is stated in the beginning. On the contrary they strongly tend to show that the present manuscript is a copy.

of Indian mathematics. There is, however, other internal evidence of unquestionable value to show that the Bakhshali mathematics cannot belong to so late a period in which Kaye would like to place it.

Bakhshali Work a Commentary

There is another noteworthy fact about the work contained in the present Bakhshali manuscript. From the method of its treatment, Hoernle thinks it to be a *Karana* work.[1] In the opinion of Datta, the Bakhshali work is not a treatise on mathematics in its true sense, but a commentary' a running commentary, of course—on such an earlier work. The manner of its composition and particularly the very elaborate, rather over-elaborated details with which the various workings of the solution are most carefully recorded, without trying to avoid even unnecessary repetitions, strongly tend to such a conclusion. Here and there are given explanatory notes of passages, literary synonyms of words and technical terms, some of which will in no way be considered difficult, or which are already well established. For instance, on folio 3 verso the word *parasparakrtam* (परस्परकृतम्) has been explained by the word *gunitam* (*tata parasparakrtam gunitam* ततपरस्परकृतं गुणितं); again on a subsequent occasion, this latter term has been interpreted as equivalent to another more difficult and less known term *abhyasa* (*tatra guna abhyasam* तत्रगुण अभ्यासं; folio 27, recto) [2]. On another occasion we have *avrttipravrttigunanam*, आवृत्ति-प्रवृत्तिगुणनं (folio 12, recto)[3]. It is stated in several instances that

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(1) Indian Antiquary, XII, p. 89.

(2) तत परस्परकृतं गुणितं जातम् (folio 3, verso);

गुणित ज्ञाता | ८ | ··· | प्रवृत्तिरित्यर्थः (folio 15, verso)

पृथक् रूपं विनिक्षिप्य। पृथक् रूपं क्षिप्तं जातं···भ्यासो तत्र गुण

| ३ | ४ | अभ्यासं | १२ | रूपहीनं | १···अभागाचतुष्पञ्चका

४५ | अत्र क्षिप्तं जातं

(folio 27, recto)

(3) तदाद्यान्तशोर्गु···ततः | ८१ / २५६ | आवृत्ति-प्रवृत्तिगुणनं ततः | ४ | अनेन गुणितं

जातं।

(folio 12, recto)

ksayampastam, क्षयंपास्तं Here is a very typical passage from the work[2].

"………Dvighnamadi' adidviguna | 2 | chayojjhitam | cha (ya) uttaram ato uttaram | patayitva ekam bhavati ….."

The above stcyle of composition is very characteristic of a commentary. And the whole work is written more or less in the same style.

Then are some cross-references in the *Bakhshali* Manuscript which are of immense significance. For instance it has been observed about the 10th *sutra* (rule), which refers to a method of multiplication, that "this rule is explained on the second page"[3] A similar remark has been made about the 14th sutra that it is "written on the seventh page"[4] The importance of these two observations in determining the character of the Bakhshali work cannot be overestimated.[5] It will be easily recognized that those observations cannot in any way be due to the author of the original treatise. For evidently those *sutras* occur at two places in the work. No author is likely to retain consciously such recurrences in his work and pass them merely by giving a cross-reference. So the duplications, as also the observations, must be attributed to a second person, the commentator. And they happened in this way : The original Bakhshali treatise was not a systematic work. It was an ordinary compon-

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(1) (i) कृत्वा रूप क्षयं पास्तमिति

—Folio 12, recto

(ii) कृत्वा रूप क्षयं पास्थमिति

—Folio 10, verso

(iii) कृत्वा रूप क्षयं पास्त | २ ३ ४ / ३ ४ ५ | जातु संगुण्य जातं

—Folio 14, recto

(2) द्विघ्नमादि । आदि द्विगुण | २ | च योज्झितं । च उत्तरं । अतो उत्तरं पातयित्वा एकं भवति……।

—Folio 50, verso

(3) एवं सूत्रं द्वितीयपत्रे विवरितास्ति

—Folio 1, recto

(4) सप्तम……पत्रेभि लिखिता…स्थि (स्ति)।

—Folio 3, rector

(5) केन कालेन सास्यतां । एवमेकादशं पत्रेऽभिलिखित पूर्वेषि

—Folio 4, verso

dium of mathematical rules and examples in which the rules relating to the same topic of discussion even, were not always put together at the same place. We notice this irregularity of treatment to a certain extent in the portion of the treatise which has been left to us, It may be pointed out that such irregular treatment is not at all unusual in case of early works and we find another instance of the kind in the *Aryabhatiya* of Aryabhata (466 A. D.). The commentator very properly attempted to improve upon the order (rather disorder) of the author, here and there, as far as posssble, without disturbing it too much, by noticing and commenting upon at the same time the *sutras which* are very closely connected. So that he had sometimes to explain a *sutra* earlier than its turn according to the plan of the author. Sometimes a commentator is compelled to refer to a subsequent *sutra* before time owing to indiscretion of the author. Therefore, when there comes the proper turn for the explanation of such a *sutra*, he simply passes it over, very naturally, by giving the cross-reference to previous pages. Thus there will remain very little doubt that the present Bakhshali work is a running commentary on an earlier work. Further there are found other cross-references which very strongly suggest that the illustrative examples are also due to the original author.[1]

Present Manuscript—A Copy

Inspite of what is stated on the contrary by Kaye[2] there are many things to make one believe that the present manuscript is not the orlginal of the Bakhshali work, but is a copy from another manuscript. For it exhibits writings of more than one scribe, possibly of five[3]. This can be explained most satisfactorily only on the

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(1) For instance, the author may give an illustrative example which may involve a mathematical principle which is yet to be explained. An instance of the kind is found in the *Trisatika* of Sridhara where the auteor very indiscretely gives two examples (Ex.7) in illustration of the Rule 16,which involves mathematical principle explained in the Rules 23 and 24. In this work the commentator, who is no other than the author of the treatise himself gives the cross-references.

(2) *Bakh, Ms.*, p. 74 footnote. "There is evidence that the *Ms* is not a copy at all. It is not the work of a single scribe : there are cross-references to leaves of the Ms: there is a case of wrongly numbering *Sutra* and the mistake is noted in another hand-writing."

—Kaye

(3) *Bakh. Ms.* p. 11, 97.

assumption that is a copy. Further on folio 4, verso, is found an observation as regards a certain *sutra* (rule) that "there is a mistake in the rule" (*sutra bhrantimasti*).* The style of writing of this observation is same as that of other writings on the leaf. So there is absolutely no doubt that all the writings on the leaf are due to the same scribe. Moreover, though this observation is placed between two lines of writings, it is not an ordinary case of interlining. From the apportionment of space in and about the remark, it is apparent that the remark was introduced at the time of making the copy, but not on any subsequent occasion. Now that observation cannot be due to the author of the original treatise. For no author would pass over a mistake in his work with a mere observation that it is wrong. So it must be from another person, possibly the scribe. There is also another possibility, and there are reasons to believe it to be more probable, that the scribe found it in the copy which he used. In any case, it will follow that the present manuscript is a copy. A more conclusive proof of this is furnished by the colophon that the work is "written (*likhitam*) by a Brahmana mathematician, son of Chajaka, for the education of the son *Vasista*."[1]. Had Chajaka been the author of the work, the more appropriate and usual word for this colophon to begin with would have been *krtam* or *viracitam* ("composed").

The scribe seems to be a careless one. For the manuscript is full of slips and mistakes. Here are a few of them :—

(1) On folio 4, verso, occurs the passage "*sodasamasutram 17*[2]. Evidently the figure should be 16.

2. On folio 8, recto, a portion, "uttarardhenabhajayet" i.

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(1) ······वशिष्टपुत्रः
सिक स्यार्थे पुत्र पौत्र उपयोग्यं भवतु: लिखितञ्छजकपुत्र गणकराज ब्राह्मणेन । सर्वेपाम्मेव शास्त्राणां गणितं मूर्ध्नि तिष्ठति । आद्यान्तसाने संसारे उत्पन्न पश्चाभृष्टि तदाकर्त्तुं शिवेन परमात्मन्᳟ याद्यं च सुत्पन्नं गणितं सख्यकारणम् ।
य च ˙ हीनं··· ··· ॥—Folio 50, recto.

(2) षोडशं सूत्रं १७—Folio 4, verso

* सूत्रभ्रान्तिमस्ति

deleted[1]. This was written by mistake for "*uttarenabhajet*" which is the relevant portion of the *sutra* meant for quotation there. The deleted portion can be traced to a preceeding *sutra* (folio 7, verso), where we have "*uttarardhena bhajitan*".

3. On folio 11, verso, 158

1

5

1

64

is twice miswritten for 158. This latter fraction is once again

13

64

wrongly written as "158 to 1 se l". Another mistake on the leaf

64

is "93 to ...masa 9;" what is meant $93\frac{3}{4}$.

(4) In the Bakhshali manuscript[2], the end of a *sutra* is usually marked by a special design. ❀ Owing to the carelessness of the scribe, the sign has been put many times at an intermediate place in the *sutra*. These illustrations are sufficient to show the carelessness of the scribe.

Exposition and Method

(i) The text consists of rules, *sutras*, and examples. There is no explanation whatever of the processes by which the rules were obtained, that is, there is no mathematical theory at all. In this the work follows the usual Indian fashion, as exhibited in all early texts. But there is a good deal of mathematical theory implied and the rules and examples are often set-forth in such a way as to convey the principles followed quite clearly to the student.

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(1) (उत्तरार्धेन भाजयेत्), this is deleted in the MS; this was written by mistake for उत्तरेण भजेत् ।
उत्तरार्धेन भाजितम्—Folio 7, verso

(2) We have denoted this design by ❀.

(ii) The rules (*sutras*) are written in verse and are generally numbered; and often in the solutions of the examples, phrases from the *sutras* are quoted. When Dr. Hoernle tried to rearrange the leaves of the manuscript, he took the numbered *sutras* as the basis of his order, but they were too few in number to lead to a satisfactory result. These *sutras* do not represent, as might be expected the most valuable part of the text. They are usually of particular application rather than general and are often very obscurely expressed.

(iii) The examplesgiven are generally formally stated in full, without the use of notation or abbreviation of any kind; and in most cases they are stated in verse. They are introduced by the term *udo**[1] an abbreviation for *udaharanam**[2] "*an example*", After the question sometimes comes a formal statement with numerical symbols and abbreviations. often arranged in cells. Then comes the solution or working (*karana*,*[3] and here, sometimes, fragments of the *sutras* are quoted. Finally comes demonstration—often more of the nature of verification than proof. Generally these demonstrations, by the aid of the answer found to the question, rediscover one of the original elements of the problem; and sometimes several such demonstrations are attached to an individual problem, but sometimes the variation is merely a matter of the form of statement.

The full scheme of exposition is therefore :

*Sutram**[4] or rule.

*Udaharnam**[5] or "*example*"; indicated its abbreviation *uda**[6]

sometimes the example is called *prasna*[1]*[7].

1. Folios 46, verso, and 65 recto.
2. Folios, 23, recto 25 verso ; 29, recto, and ,55, verso. Compare also folio 35, recto (a) and verso (b).
3. Folios 32, 36 44, verso and 46, recto. These references must have escaped the notice of Hoernle who remarks otherwise.

*१ (उद); *२ (उदाहरणम्); *३ (करण;) *४ (सूत्रम्); *५ (उदाहरणम्); *६ (उद); *७ (प्रश्न) ।

Sthapanam or "*statement*". Sometimes it is also called *nyasa*[2]*1 or *nyasa-sthapana*[3]*2.

*Karanam**3 or "solution".

*Pratyayam**4 or "verification". (Some times a solution is verified in more than one way.

The end of such *sutra* is marked after last example by the device and the number of the *sutra* is also given at the end.

The above will explain in general the method of the exposition of the Bakhshali work. But there are also occasional deviations from it. For it is not always that a *sutra* is illustrated by examples and an example is followed by its solution. There are at least two *sutras* in the surviving portion of the Bakhshali work which have no examples attached to them. They have been passed over as having been explained or written on preceeding pages. Two examples are left without solution with similar remarks. Again solutions of examples in the beginning of the work have got no verifications, and so also a few others[3] .

The above method of exposition differs considerably from what is now commonly met with in other Hindu mathematical treatises.

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(1) For exauple, on folio 11, verso, there is mention of verification by the fourth method (anvam caturthapratyayam krlyante)

(2) Vide folios 1—3: also folios 23—25.

(3) *BrSpSi.* ed. by Sudhakara Dvivedi Benaras, 1912, Solutions of some of the examples have been included as authentic in Colebrooke'- *Siddhanta* (H. T. Colebrooke, Algebra with Arithmetic and Mensuration from the Sanskrit of Brahmagupta and Bhaskara, London, 1817) translation of the arithmetical and algebraical portion of Brahmagupta's. But they are not found in Dvivedi's edition of the work. Dvivedl also procured for himself a transcript of the copy of the commentary by Prthudakasvami which was originally secured by Colebrook (vide Dvivedi's Preface, p. 3). In these circumstances, we accept Dvivedi's version of Brahmagupta's above mentioned work to be more authentic. So it wiil have to be said that Brahmagupta did not give any solut on of his examples. See also the *Brahmasphuta-Siddhanta*, New Delhi, 1968.

*१ (न्यास); *२ (न्यासस्थापना); *३ (करणम्); *४ (प्रत्ययम्) ।

Brahmagupta (628 A.D.) gives a very few examples (*udaharana*) in illustration of a limited number of his rules, but not their solution[2]. This want has been amply made up by his eminent commentator *Prthudakasvami* (860A.D.) who has supplied sufficient number of illustrative examples with soluiions uuder each rule. Mahavira(cd.850A.D.) gives a copious number of examples for each rule[1]. He calls them *uddesaka**. But he too does not give solution The first writer to give statements (*nyasa*) as well as answers of his examples called *udaharana* is Sridhara (c. 750 A. D.)[2]. Then comes Bhaskara[3]. These writers have not recorded working of their solution. And no other Hindu writer is known to have given any verification of the solution of their examples. We do not find this even in the works of the latter commentators.

(iv) The method of grouping sets of figures is of interest, and shows features in common with mediaeval Sanskrit mathematical manuscripts, where also it is the practice to place groups of numbers in cells. In the Bakhshali manuscript, however, this fashion is rather more elaborated than in any Sanskrit manuscript so for examined. The mathematical possibilities of this scheme do not appear to have been realized and the student must always be careful to interpret any group of figures from the context, and not from any similarity with other groupings. However, there is a certain amount of consistency in the arrangements, as the examples exhibited below will show. The real purpose of the arrangements appears to prevent confusion by demarcating the numerical figures from the text itself. The text is

(1) GSS., edited with English translation by T. Rangacharya Madras 1912.

गणित सारसंग्रह

(2) *Trisatika*, edited by Sndhkara Dvivedi, Benaras, 1899.

त्रिशतिका

(3) *Lilavati* edited by Sudhakara Dvivedi, Benaras, 1910, *Bijaganita*, edited by Sudhakara Dvivedi and revised by Muralidhara Jha, Benaras, 1927. English translation of these works are incorporated in Colebrook's Hindu Algebra,

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उद्देशक

often written almost independently of the figure groups, and a word may be arbitrarily divided by the cell araangement, which may also cut into several lines of the text not necessarily connected with it. The economic neeessity of utilizing the whole of the writing surface of the birch-bark availabe seems to have been the determining factor. The text itself should be consulted but the following examples may be helpful :

(a) Integral numbers occasionally occur without any marking off by lines or cells, but often.

(b) each integral number has a cell to itself, e. g.

| 1 | | 42 | | 39 |

(c) Sometimes an integer is marked off by two vertical bars : thus | 14 | and invarlably a series of integers is thus demarcated, e. g.

| 20 | 40 | 60 | 80 | evam 200 |

(d) Fractions and groups of fractions are placed in cells or groups of cells, e. g.

(i) | 132 | over | 33 | (ii) | 6055040625 | over | 3227520000 | (iii) | 1 1 1 1 | over | 4 3 6 12 |

(iv)
| 1 |
| 1 |
| 3 |
| 1 |
| 4 |
| 1 |
| 5 |

(v)
| 40 |
| 1 |
| 3+ |
| 1 |
| 4+ |
| 1 |
| 5+ |

(vi)
| 21 |
| 20 |
| 21 |

(vii)
| 7 |
| 12 |
| 12 |

(e) Complete sets of operations are sometimes marked off in a similar manner. For example, (d) (iii) means [illegible] (d) (iv) means $(1+\frac{1}{3})\ (1+\frac{1}{4})\ (1+\frac{1}{5})$; (d) (v) 40 $(1-\frac{1}{3})$ [illegible].

(f) Series of operations may be connected together by cell arrangments; for example :

2 di° 1 2	1 di° 1 2	100000 di° 947	phalam di° 60000 947
3 di° 1 2	1 di° 1 3	157500 di° 947	phalam di° 60000 947
4 di° 1 2	1 di° 1 4	216000 di° 947	phalam di° 60000 947

which means,

$$2\frac{1}{2} \text{ dinaras} :: 1\frac{1}{2} \text{ days} :: \frac{100000}{947} \text{ dinaras} : \frac{60000}{947} \text{ days}$$

$$3\frac{1}{2} \quad ,, \quad : 1\frac{1}{3} \text{ days} :: \frac{157600}{947} \quad ,, \quad : \frac{60000}{947} \quad ,,$$

$$4\frac{1}{2} \quad ,, \quad : 1\frac{1}{4} \quad ,, \quad :: \frac{216000}{947} \quad ,, \quad : \frac{60000}{947} \quad ,,$$

(g) The data of the problem and the solution may be indicated in one combined statement; e.g.

si° 1 1	16 1 1 4 1 3	4 a° che° 2187* 1 1	phalam sara 2624400

This is a statement of properties where the second term means

1⌐ $(1+\frac{1}{4})$ $(1+\frac{1}{3})$ and the number marked with an asterisk is a charge-ratio.

$$\text{(h)} \left| \begin{matrix} 0 \\ 1 \end{matrix} \right| \begin{matrix} 2 & 1 & 3 & 3 \\ 1 & 1 & 1 & 1 \end{matrix} \left| \begin{matrix} 12 & 4 \\ 1 & 1 \end{matrix} \right| \quad \text{dr}^{\circ} \quad 300$$

may be roughly expressed by means of $x + 2x + 3 \times 3x + 12 \times 4x = 300.$

(v) The use in our text of the sexagesimal notation in the form in which it occurs is of rather special interest, for there is, as far as we know no other example of the kind in any of the classical Sanskrit works. The Hindus, from Aryabhata onwards, were well aware of the advantages of the sexagesimal notation for astronomical purposes. but they never used it for an arithmetical purpose.

Apparently there is only one purely arithmetical example of the use in the text and this example occurs, in connection with a problem in arithmetical progression, on folio 6, verso, and 7. recto, where the fraction 178/29 is expressed as $6 + 8' + 16'' + 33''' + 6^{iv}$. This sexagesimal fraction is actually written thus—

6
8
60
16 cha°
60
33 li°
60
6 vi°
60
se° 6
29

The upper three figures are missing in the manuscript but the restoration is certain. Of the abbreviation, li° stands for *lipta*, लिप्त (Gk. *lipte*) which in Sanskrit words ordinarily means a minute of arc, or the sixtieth part of a degree; vi° stands for *vilipta* विलिप्त, ordinarily a second of arc : while *se*° stands for *sesam*, शेषम् or "remainder".

Here one may not agree with Kaye that the purely arithmetical (and perfectly legitimate) use of the notation points to extra-India influence (Kaye thinks that although such a use is unknown in Sanskrit works, it was extremely common in mediaeval Muslim works).

It will be noticed that the term *lipta* here applies to "third parts instead of 'first parts', and *vilipta* to 'fourth parts'. The abbreviation *cha*° has not yet been traced to its origin.

(vi) The modern place-value arithmetical notation is employed throughout the text and Kaye says that there is not the slightest indication that to the author, it was a new or strange invention.

(vii) As already indicated algebraic symbols are not generally employed, but a symbol for the unknown quantity is. This symbol is the arithmetical symbol for *'nought'* or *'zero'* and on several occasions it is referred to by the term *sunya-sthana* (शून्यस्थान) or 'empty place'.

The symbol occurs some twenty times but only in sections B, C, F, G, H, K. Its employment is illustrated in the following exmple ;—

(a)

a°	1 1	u°	1 1	pa°	0 1	labdham	10 1

This is a 'statement' of an arithmetical progression where the first term is 1, the common difference is 1, the number of terms unknown, and the 'quotient' is 10 (where $10x$ = the sum of the series). Here the symbol simply indicates that the number of terms *pada* is unknown, i.e., that the place in the statement is empty. The symbol does not enter into any operation here or elsewhere. The giving it a denominator of unity is curious and really indicates that it is an integral number.

(b)

a°	5 1	u°	6 1	pa°	0 1	dha°	0 1
a°	10 1	u°	3 1	pa°	0 1	dha°	0 1

Here are two equivalent arithmetical progressions in which the number of terms are equal and the sums also are the same in both cases, but both are unknown.

(c)	0	1	1	1	1	bha°	se	16
	1	1	1	1	1			1
		3	3	3	3			

which means $x=16/(1-\frac{1}{3})\ (1-\frac{1}{3})\ (1-\frac{1}{3})\ (1-\frac{1}{3})$ and since this is a deduction from a problem it is a distinct step towards a proper algebraic symbolism.

(d)	0	2	3	4	drsya	200
	1	1	1	1		1

which may be represented by $x+2x+3x+4x=200$ But the method of solution is by the *regula falsi.* Any number is put in the place of the 0, and the sum of the series is obtained. Here the given sum multiplied by the assumed value and divided by the (false) sum is the correct solution...

(e)	0	5	yu	mu	0	sā	0	7	mu	0
	1	1			1		1	1		7

which means $x+5=s^2$, $x-7=t$. Here the symbol 0/1 stands for three different unknown quantities. It simply indicates in each case an unknown number.

(viii) *Negative Sign* : The only distinct mathematical symbol employed is the sign for the negative quantity, which takes the form of a cross + placed after the number affected. This is peculiar and has given rise to discussions. In Sanskrit manuscript a dot before the quantity affected is the usual method of indicating a negative quantity. In the transliteration of the text the original negative sign (+) is, perhaps illogically preserved; but it is a rather special feature of the manuscript, easily printed, and leading to no ambiguity.

On this negative sign Dr. Hoernle writes [1]

(1) *Indian Antiquary,* XVII (1888), p. 34.

"Here, therefore, there appears to be a mark of great antiquity As to its origin I am unable to suggest any satisfactory explanation I have been informed by Dr. Thibaut of Benares that Diophantus in his Greek arithmetic uses the letter ψ (short for *elleipsis*) reversed (thus ⋔) to indicate the negative quantity. There is undoubtedly a slight resemblance between the two signs; but considering that the Hindus did not get their elements or the arithmetical science from the Greeks (2), a native Indian origin of the negative sign seems more probable. It is not uncommon in Indian arithmetic to indicate a particular factum by the initial syllable of a word of that import subjoined to the terms which compose it.......... ..The only plausible suggestion I can make is, that it is the abbreviation (*ka*) of the word *kanita*, diminished".

(ix) *Abbreviations* :—Abbreviations are employed to such an extent as to become, at times embarrassing, and they embrace almost every type of term. None of these abbreviations is used in an algebraic sense, although, at first sight, when we find a°, u°, pa° used consistently for the elements of an arithmetical progression, an algebraic symbolism does not seem very far off. But a°, ha°. u°, go° are the abbreviations of names of animals ya°, go° and sa° of plants, etc., ect. The names of measures which are numerous, are nearly always abbreviated. Certain common terms of operation are often abbreviated, and of these the following occur most often—

bha[1] for bhaga[2], placed after a term to indicate that it is a divisor.

se°[3] for sesam[4], a remainder.

mu°[5] for mulam[6], a root, a quantity that has a root, capital.

pha°[7] for phalam[8], an answer.

etc.. etc.

Indeed the writers seem to have used abbreviations whenever there was no ambiguity incurred in their own minds. We are not so fortunately placed in this matter and occasionally it has been impossible to rediscover the term implied.

(2) They got a good deal from them, and particularly all their later astronomy came from the Greeks.

१ (भ) २ (भाग) ३ शे) ४ शेषम्) ५ (मू) ६ (मूलम्) ७ (फ) ८ (फलम्)

A 1 Illustration

Just to show how the problems are treated in the Bakhshali Manuscript, we shall reproduce one il ustration:

(Folio I, verso and II, recto).

The *sutra* means change $\frac{a}{b}$ to $\frac{b}{b+a}$ and quotations from it are given on Folios 1, verso and 2, verso. The example solved on 1 verso and 2 recto is somewhat as follows :

The combined capitals of five merchants less one-half of that of the first, one-third that of the second, one-fourth that of the third one-fifth that of the fourth, or one-sixth that of the fifth is equal to the cost of a jewel. Find the cost of jewel. and the capital of each merchant[1]

---

(1) दशम सूत्रम् 10

सूत्रम्—।। अंशां विशोद्ध्यच्छेदेभ्य कुयति् तत्परिवर्तनम् ।।

···सास्यं तत प्रोज्य धनान्विश विनिर्दिशेत् ।

उदाहरण —।। पञ्चानां वणिजा मध्ये मणि विक्रीयते किल

तत्रोक्ता मणि विक्रीता मणि मूल्यां क्रियद् भवेत्

... ... ···दम्

अर्धं त्रिभाग पादांशं पञ्च भाग षोडश च

···ततो प्रोज्य: सदृशं क्रियते···जाता

120	90	80	75	72	तत्र प्रोज्य जातं 1.0	90	80	75	72
60	60	60	60	60					

एषां योग कृते जात 427 अतो शेष 377 एष मणि मूल्यम् ।

चतुर्थीसं क सर्वस्वम् ।।

प्रथमस्य सांक अर्धम् ··90 | 80 | 75 | 72 चतुर्णाम् योग 317 प्रथमार्धेन षष्टिभिर्युतं 377 एष प्रथमस्य धनं प्रथम धनं ।

तृतीय चतुर्थं पञ्चमस्य धनं सर्वस्वम् 347 द्वितीय त्रिभागं 30 एष युतं 377 एष द्वितीयस्य धनं भवति ।

contd. on next page

Verso 1 appears to give part of the solution and proofs of the question on Folio 1, recto. Since.

$$< \Sigma x-\tfrac{1}{2}x_1 = \Sigma x-\tfrac{1}{3}x_2 = \Sigma x-\tfrac{1}{4}x_3 = \Sigma x-\tfrac{1}{5}x_4$$
$$= \Sigma x-\tfrac{1}{6}x_5 = C,$$

We have $\tfrac{1}{2}x_1 = \tfrac{2}{3}x_2 = \tfrac{3}{4}x_3 = \tfrac{4}{5}x_4 = \tfrac{5}{6}x_5 = k>$

$$\text{Whence} \quad \sum x = \frac{120+90+80+75+7}{60} = \frac{437}{60}k$$
$$= \frac{437C}{437-60}$$

पुन: प्रथम द्वितीय चतुर्थ पञ्चम···सर्वस्वं 357 तृतीयस्य पार्द 20 एव युक्तं 377, एष तृतीयस्य धनं भवति ।

पुनरपि प्रथम द्वितीय तृतीय पञ्चमस्य 362 चतुर्थस्य पञ्चोभाग 15 एव युतं 377, एष चतुर्थस्य धनं भवति *Folio 1 verso.*

अथ प्रथ···त्यंषष्ठि शेषम् 377

अथ द्वितीयस्य	120 30 80 75 12	एवं 377 द्वितीयस्य धनं भवति
अथ तृतीयस्य क्रियते	120 90 80 75 70	एवं 377 तृतीयस्य धनं भवति
चतृर्थस्य क्रियते	120 90 80 15 72	एवं 377 चतुर्थस्य धनं भवति
पञ्चमस्य क्रियते स्थापनम्	120 90 80 75 12	एवं पञ्चमस्य 377

(Folio 2, recto)

If $k=60$, then $C=377$, and $x_1=120$, $x_2=90$, $x_3=80$, $x_4=75$ and $x_5=72$.

Then follows a proof which may be expressed,

$$90+80+75+72=317 \text{ and } 317+\tfrac{120}{2}=377;$$

$$120+80+75+72=374 \text{ and } 347+\tfrac{90}{3}=377;$$

$$120+90+75+72=375 \text{ and } 357+\tfrac{80}{4}=377;$$

$$120+90+80+72=362 \text{ and } 362+\tfrac{75}{5}=377;$$

$$120+90+80+75=365 \text{ and } 365+\tfrac{72}{6}=377$$

Again on Folio II, recto, there appears to be another verification of the example which we had on I, recto and verso.

$$\tfrac{120}{2}+90+80+75+72=377$$

$$120+\tfrac{90}{3}+80+75+72=377$$

$$120+90+\tfrac{80}{4}+75+72=377$$

$$120+90+80+\tfrac{75}{5}+75=377$$

$$120+90+80+75+\tfrac{72}{6}=377$$

and this is the measure of the price of the jewel.

Fundamental Operations

In the classical Sanskrit works there is generally very little formal information about mathematical principles and methods. Axioms or postulates seldom or never occur and strict definition is seldom attempted. Rules are of particular application rather than general · order appears to have been a matter of convenience rather than logic; and the fundamental operations receive scanty attention.

(i) Examples of *addition* (1)

(a) 960 | 64 yutam jatam 1024

(b) $\left|\begin{matrix}10\\3\end{matrix}\right|$ sa rupam $\left|\begin{matrix}13\\3\end{matrix}\right|$

(c) | 2 | rupa samyutam | 3 |

(d) 840 | 49 | datva jatam 889.

(e) 924 | 836 | 798 | esham yutim kriyate 2558

(f) 120 | 90 | 80 | 75 | 72 | esham yoga krite jata 437.

(g) | 120 90 80 75 12 | evam 377

(h) | 10 | 30 | 90 | ekatram | $\frac{130}{1}$ |

(i) | 29 48 58 | 7 anena yutam | 36 48 58 |.

(k) | 5 she 1 / 1 16 | | 10 se 15 / 16 | evam 16

(l) $\frac{45}{2}$ sardha traya yutam $\frac{52}{2}$

(ii) Examples of *subtraction* (ii) :

(a) | 5 | 9 | vishesham | 4 |

(b) | $\frac{3}{2}$ $\frac{2}{1}$ | vishesham | $\frac{1}{2}$ |

---

(1) (a) 960 and 64 added : 1024 is produced. (b) $\frac{10}{3}$ plus unity $=\frac{13}{3}$. (c) 2 and unity added together = 3. (d) 49 having been given to 840, 889 is produced. (e) 924, 836, 798 : the sum of these may be determined: they produce 2558. (f) 120, 90, 80, 75, 72 ; the sum of these is made : they produced 437. (g) 120, 90, 80, 75, 12 : thus 377. (h) 10, 30, 90 : altogether 130,
(i) $29\frac{4}{5}\frac{8}{8}$ by this (7) increased = $36\frac{4}{5}\frac{8}{8}$. (k) $5\frac{1}{16}+10\frac{15}{16}$ thus 16. (l) $\frac{45}{2}$ with three and a half added = 52/2.

(ii) (a) 5, 9 : the difference is 4. (b) $\frac{3}{2}$, 2 : the differcnce is 1/2, (c) 5, 3 : subtracted, 2 is produced; (d) 6, 3 : the difference is 3; (e) 42 less three is 39; (f) 3, 7 : having subtracted, 4, (g) $\frac{77}{11}$, $\frac{294}{11}$: having subtracted the difference is $\frac{217}{11}$.

(c) | 5 | | 3 | rahitam jatam | 2 |

(d) 6 | 3 | shuddhi | 3 |

(e) | 42 | tryunam | 39 |

(f) | 3 | | 7 | vishodhya | 4 |

(g) | $\frac{77}{11}$ | | $\frac{294}{11}$ | patya shesham | $\frac{217}{12}$ |

(ii) Examples of *multiplication* :

(a) | 2 | dvigunam | 4 | , i. e., $2 \times 2 = 4$.

(b) | 30 | asta gunam | 240, i.e. $30 \times 8 = 240$,

(c) | $\begin{matrix} 2 & 40 \\ 5 & 1 \end{matrix}$ | gunita jatam 16, i. e. $\frac{2}{5} \times \frac{40}{1} = 16$.

(d) | $\begin{matrix} 4 & 1 \\ 1 & 1 \\ 2 & 4+ \end{matrix}$ | gunita jata | $\frac{27}{8}$ | i. e. $(4+\frac{1}{2})\ (1-\frac{1}{4}) = \frac{27}{8}$.

(e) | $\begin{matrix} 6 & 1 \\ 1 & 1 \\ & 4+ \end{matrix}$ | anena gunitam jatam | $\begin{matrix} 4 \\ 1 \\ 2 \end{matrix}$ |

i. e. $6 \times (1-\frac{1}{4}) = 4\frac{1}{2}$

(f) | $\frac{2}{3}$ | $\frac{3}{4}$ | $\frac{4}{5}$ | gunita jatam | $\frac{2}{5}$ |

i. e. $\frac{2}{3} \times \frac{3}{4} \times \frac{4}{5} = \frac{2}{5}$

(g) | $\begin{matrix} 40 \\ 1 \\ 1+ \\ 3 \\ 1 \\ 4+ \\ 1 \\ 5+ \end{matrix}$ | phalam 16, i.e. $40(1-\frac{1}{3})\ (1-\frac{1}{4})\ (1-\frac{1}{5}) = 16$

(h) | 3 | 4 | abhyasam 12, i.e., $3 \times 4 = 12$.

(i) | 8 | atma-gunam 64, i. e. $8 \times 8 \times 64$.

(j) $\begin{array}{|c|c|}880 & 964\\ 84 & 168\end{array}$ gunita jatam $\begin{array}{|c|}848320\\ 14112\end{array}$

i. e. $\frac{880}{84}\times\frac{964}{168}=\frac{848320}{14112}$

(k) $\begin{array}{|c|c|}737 & 178\\ 58 & 29\end{array}$ anena gunitam jatam $\begin{array}{|c|}65593\\ 841\end{array}$

i. e. $\frac{737}{58}\times\frac{178}{29}=\frac{65593}{841}$

(l) $\begin{array}{|c|}108625\\ 65600\end{array}$ pada-ghna $\begin{array}{|c|}6455040625\\ 3227520000\end{array}$

$\frac{108625}{65600.}$ multiplied by number of terms $\begin{array}{|c|}59425\\ 49200\end{array}$

$=\frac{6455040625}{3227520000}$

(m) $\begin{array}{|c|c|}405280 & 44004\\ 38724 & 77448\end{array}$ samgunya jatam $\begin{array}{|c|}179945781120\\ 2999096352\end{array}$

i. e., $\frac{405280}{38724}\times\frac{44004}{77448}=\frac{179945781120}{2999096352}$

(iv) *Examples of Division* : Fraction quantities are expressed in the usual Indian way with the numerator written above the denominator with out any dividing line, and they are usually placed in cells. This indic ition of the operation of division often does away with the necessity for an explanatory word or phrase.

Examples of division :

(a) $\begin{array}{|c|c|c|}168 & 168 & 168\\ 4 & 6 & 7\end{array}$ labdham 42 | 28 | 24 |

i.e. $\frac{168}{4}=42;\ \frac{168}{6}=28;\ \frac{168}{7}=24$

(b) $\begin{array}{|c|}4\end{array}$ vibhaktam $\begin{array}{|c|}1\\ 4\end{array}$ 35 gunitam $\begin{array}{|c|}35\\ 4\end{array}$

i.e. 4 divided $=\frac{1}{4}$, which multplied into $35=\frac{35}{4}$

(c) $\begin{array}{|c|c|}10 & 3\end{array}$ vibhaktvam $\begin{array}{|c|c|}10 & 1\\ 3 & \end{array}$

i e. 10, 3 having divided $=\frac{10}{3}$

(d) $\begin{array}{|c|}447\\ 29\end{array}$ dalita $\begin{array}{c}447\\ 58\end{array}$

i.e. $\frac{447}{29}$ halved $=\frac{447}{58}$

(e) $\begin{array}{c|}132\\ 33\end{array}$ vartyam jatam $\begin{array}{|c|}4\\ 1\end{array}$

i,e. $\frac{132}{33}$ redused gives 4,

(f) | 798 / 1463 | projjhya 798

i e. having described the denominater $\frac{798}{1463}$ becomes 798

(g) 2558 chchheda projjhyam 1095

i.e, $2558-1463=1095$, where 1463 is the denominator of the fraction.

(h) | 60 anena drishyam bhajitam | 1/60 300/1 | jeta | 5 |.

i.e. by this (60) the known quantity (300) is divided and $\frac{1}{60}$ of $300=5$

Square-root and Surds

The method of extracting square-roots of non-square numbers exhibited in the text is of special interest. The rule is much mutilated but as it is given on three separate occasions the fragments pieced together enable us to give the complete *sutra* [1]

"In case of a non-square (number), subtract the nearest square number; divide the remainder by twice (the root of that number). Half the square of that (that is, the fraction just obtained) is divided by the sum of the root and the fraction[2] and substract; (this will be the approximate value of the root) less the square of the last term."[2]

---

(1) The rule is not preserved in its entirety at any one place in the mutilated manuscript of the Bakhshali mathematics that has come down to us. But from references and quotations one can restore the rule completely :

| 21 |
| 20 |
| 21 |

अकृते श्लिष्ट कृत्यूना शेषच्छेदो द्विसंगुण: ।
तद्वर्ग. दल संश्लिष्ट: हृति शुद्धि कृति क्षय:

Folio 56, recto

अकृते श्लिष्ट कृद्यूना शेषच्छेदो द्विसंगुण:

Folio 57 verso.

(2) English translation of this rule given by Kaye (Bakh. Ms Sec. .68) is discarded as being wrong and meaningless. He has admittedly failed to grasp the true significnce of the text. "The rule as it stands," observes Kaye, "is cryptic and hardly translatable." He further thinks that "no rule for 'second approximations' is preserved". It is not true, The method of

(foot-note carried to next page)

Then follows a note to the effect that "by means of this rul an approximation (अनय) to the proper root of a mixed quantity is found. . . . .".

The rule as it stands is cryptic and hardly translatable, but fortunately there are examples given in some detail, and these show that the rule was extended to "second approximations".

The rule means that the first approximation to

$\sqrt{Q}=\sqrt{A^2+b}$ is $A+b/2A$ or q_1 ; but $q_1^2-Q=(b/2A)^2$
$= e_1$.

No rule for 'second approximation' is preserved but there are several examples; and, of course, no fresh rule is really required, for $\sqrt{Q} = \sqrt{q_1^2 - e_1} \sim q_1 - e_1 / 2q_1 =$

$A + \frac{4A^2 + b^2}{8A^3 + 4Ab}$ and the 'second error' may be indicated by,

$$e_2 = \left[\frac{e_1}{2q_1}\right]^2 = \left[\frac{b}{2A}\right]^4 / 4\left(A + \frac{b}{2A}\right)^2$$

All the examples preserved belong to section 'C' and appear to be merely subsidiary to the solution of certain quadratic equations

---

finding the third approximation is indicated in the second line of the verso. Indeed the only two difficult words in the *sutra* are samshlista (संश्लिष्ट) and kritikshaya (कृतिक्षय) We have interpreted the form as referring to "the sum of the root and the fraction," That that is the interpretation aimed by the author will be corroborated by what is written in the folio 56, recto; while evaluating the surd $\sqrt{481}$.

21					21	
20	400	दल)	1	(विश्लिष्टः)	20	(भा)
21	441		2		21	

Compare also folio, verso. Kriti-kshya (कृतिक्षय) literally means "loss of the squares" or less the "square". It means that the square of the approximate value thus found less the square of the last term of it will be equal to the given quantity. The commentator has attempted an explanation of it on folio 57, recto but it is much mutilated. He says: kritikshayam kritam esamulam (कृतिक्षय कृत एषमूल). Our explanation seems to be the only rational one possible.

arising out of, problems in arithmetical progressions. These problems have been fully worked out by Kaye in Sec.68 and below are given merely a summary of the square-root evaluations. There are no less than six such applications of this rule in the Bakhshali manuscript.[1]

(i) $\sqrt{41} = \sqrt{36+5}$ and $b_1 = 6\frac{5}{12}$ while $e_1 = \frac{45}{144}$

and $q_2 = 6\frac{745}{1848}$

(ii) $\sqrt{105} = \sqrt{100+5}$ and $q_1 = 10\frac{1}{4}$; $e_1 = \frac{1}{16}$;

$$q_2 = 10\frac{1}{4} - \frac{\frac{1}{16}}{2\times 10\frac{1}{4}} = 10\frac{81}{328} \;;\; e_2 = \left(\frac{\frac{1}{16}}{2\times 10\frac{1}{4}}\right)^2$$

$$= \frac{1}{107584}$$

(1)* Folios 57 and 64, verso; 45, recto; 56, recto and 56, verso: 45 and 46, recto. Note the expression mulam shlistakaranya (मूलं श्लिष्टकरण्या) or "the root by the method of approximation" folio 65, verso).

64 verso, 57, verso, and 57 recto: are all (except the last line) concerned with one example, the beginning of which is lost. The example is $\left[a = 1, d = 1, s = 5\right.$; therefore,

$\left. i = \frac{\sqrt{41}-1}{2}\right]$. The first approximation to $\sqrt{41}$ is

$q_1 = 6\frac{5}{12} = \frac{77}{12}$ and $t_1 = \frac{65}{24}$. Therefore.

$$s_1 = \left[\left(\frac{65}{24} - 1\right)\frac{1}{2} + 1\right]\frac{65}{24} = \left(\frac{1}{2}\cdot\frac{41}{24} + 1\right)\frac{65}{24} =$$

$$\frac{89}{48}\cdot\frac{65}{24} = <\frac{5785}{1152} = 5\frac{25}{1152} = 5 + \frac{1}{8}\left(\frac{5}{12}\right)^2>.$$

The second approximation is introduced by the square-root rule (as previously on folio 55, recto) and is given by :

$$q_2 = 6\frac{5}{12} - \frac{1}{2}\left(\frac{5}{12}\right)^2 \Big/ 6\frac{5}{12} = \frac{11858-25}{1848} = \frac{11833}{1848}.$$

and $t_2 = \frac{1}{2}\left(\frac{11833}{1848} - 1\right) = \frac{1}{2}\cdot\frac{9985}{1848} = \frac{9985}{3696}$ and

"this is the number of terms when the sum is five)" —Kaye, *Bhakh*, *Ms.* Part III, p. 181.*

(iii) $\sqrt{481} = \sqrt{21^2+40}$; $q_1 = 21\frac{40}{42}$; $\frac{e_1}{8} = \frac{1}{8}\left(\frac{40}{42}\right)^2$

$$= \frac{1600}{14112};$$

$$q_2 = 21\frac{20}{21} - \left(\frac{20}{21}\right)^2 \Big/ 2 \times 21\frac{20}{21}$$

$$= 21\frac{9020}{9681}\ ;\ \frac{e_2}{8} = \frac{1}{8}\left(\frac{40}{42}\right)^4 \Big/ \left(2 \times 21\frac{40}{42}\right)^2$$

$$= \frac{160,000}{2,999,096,352}.$$

(iv) $\sqrt{889} = \sqrt{29^2+48}$; $q_1 = 29\frac{48}{58}$; $\frac{e_1}{24} = \frac{1}{24}\left(\frac{48}{58}\right)^2$

$$= \frac{24}{841}.$$

(v) $\sqrt{336009} = \sqrt{579^2+768}$; $q_1 = 579\frac{384}{579}$;

$$q_2 = 579\frac{515,225,088}{777,307,500};$$

$$\frac{e_2}{8d} = \frac{21,743,271,936}{7,250,483,394,675,000,000}.$$

(vi) $\sqrt{339009} = 579 + \frac{384}{579} - \frac{\left(\frac{384}{579}\right)^2}{2\left(579+\frac{384}{579}\right)}$

There is not much doubt about the exegesis of the rule. It was neither connected with continued fractions, nor with the so called Pellian equation. Brahmagupta gave the converse of the rule, namely $(A+x)^2 \backsim A^2+2Ax$ from which the square-root rule given in our text is immediately deducible. In this connection, Kaye has been of opinion that the square-root rule itself was not used by the Indians and was not even noticed by them until the sixteenth century. Indeed the Hindus had a very good practical rule of their own, which was given by Sridhara (*Trishatika*, 46) and Bhaskara II

(*Lil ivati*, 138), namely—"Multiply the quantity whose square-root cannot be found by any large square number, take the square-root of the product—leaving out of account the remainder—and divide by the square-root of the multiplier". For example, $\sqrt{41} = \sqrt{41 \times 1000000} \div 1000 \sim 6.403+$

The above approximate formula is now generally attributed to the Greek Heron (c. 200 A.D.)[1], and it is restated by the Arab Al-Hassarar (c. 1175 A.D.) and other mediaeval algebraists[2]. But it was known, as has been shown elsewhere, to the second order of approximation, to the ancient Indians several centuries before[3].

The values of these surds are summarised below :

Q	q_1	q_2	q (approximately correct).
41	6.41667	6.40313	6.403124
481	21.9524	21.9307	21.9317122
889	29.828	—	29.8161030
33609	579.664	57966615	579.65···

We have in the '*Sulba sutras* (*Baudhayana*, c 800 B. C.) the value of $\sqrt{2}$, and the other surds, not only obtained by geometric devices but also by the process of continued fraction. Kaye is awefully mistaken in asserting that the "Square-root" rule was not used by the Hindus and was not even noticed by them until the sixteenth century.

Rule of Three

The term *trairasika**[1] "relating to three quantities" occurs in the Bakhshali Manuscript about a dozen times—always employed in the sense of arithmetical proportion. Most often it occurs in the

(1) T. Health, "*History of Greek Mathematics*", Vol. II, p. 324 ; Heron's time is uncertain. He may have lived in the 3rd century. A. D.

(2) D. E. Smith, History of Mathematics" in two volumes, 1972, Vol. II, p. 254.

(3) Bibhutibhusan Datta "*Hindu Contribution to Mathematics*", Bulletin of the Mathematical Association, University of Allahabad, (1927-28), 269),

phrase *pratyaya trairasikena**3 or "proof by the rule of three" and the proportion is generally set out in the following manner,

1	1	4	फलम्	
3	1	1	phalam	18
	2			1

which means $\frac{1}{3} : 1\frac{1}{2} : : 4 : 18.$

The term *trairasika* is orthodox but the term *phalam**3 is used throughout our text quite appropriately as equivalent to "answer", and is not applied to the second term of proportion as in the *Lilavati**4.

There are very many examples of the 'rule of the three' used either as the direct method of solution or as a proof. There are also some very much damaged examples of what appears to be problems in so-called compound proportion. There is nothing of the nature of a theory of proportion discussed but several examples of the following principle occur—

If,

$$\frac{a_1}{b_1} = \frac{a_2}{b_2} = \cdots = \cdots = \frac{a_n}{b_n}, \text{ then } \frac{\Sigma a}{\Sigma b} = \frac{A}{B}.$$

Calculation of Errors and Process of Reconciliation

The Bakhshali mathematics exhibits an accurate method of calculating errors and an interesting process of reconciliation, the like of which are not met elsewhere. They are necessitated by the application of the foregoing approximate square-root formula. There are certain examples whose solution leads to the determination of the number of terms of an arithmetical progression whose number of terms is unknown but first term (a), common difference (d), and the sum (s) are known. If the number of terms be t, then according to the Bakhshali work,

$$s = \left[(t-1)\frac{d}{2} + a\right] t, \qquad \ldots(1)$$

6

whence $t = \dfrac{-[2a-d]+\sqrt{[2a-]d^2+8ds}}{2d}$

The negative sign of the radical has been overlooked in the Bakhshali work. Putting $p = 2a-d$ and $Q = [2a-d]^2+8d$, we have,

$$t = \frac{-p+\sqrt{Q}}{2d}$$

$$\left.\begin{aligned} &\text{or} \quad 2dt+p = \sqrt{Q} \\ &\text{also} \quad 2s = t^2d+pt \end{aligned}\right\} \qquad \ldots(2)$$

Oftentimes in the examples given the value $\sqrt{Q}$ does not come out in exact terms, so that a method of approximation has to be adopted. Let q_1, q_2........,be the successive approximations to the value of $\sqrt{Q}$ and let the values of t obtained from them be t_1, t_2, ...Neither of these values will evidently give the original quantity s when substituted in the equation [1] for the purpose of verification of the results obtained. Suppose the values of s corresponding to the values of t be s_1, s_2......

Then,

$$2s_1 = t_1 2d + pt_1$$

or $\quad 8ds_1 + p^2 = \left(2t_1d+p\right)^2$

and $\quad 8ds + p^2 = (2td+p)^2$

Therefore.

$$8d(s_1-s) = (2t_1d+p)^2 - (2td+p)^2$$

now, $\quad 2td_1 + p = q_1$

(1) The *Kriti-ksaya* (कृतिक्षय) probably refers to this second error.

Hence, $s_1 - s = \dfrac{q_1^2 - Q}{8d}$

Since $\sqrt{Q} = \sqrt{a^2+r} = a + \dfrac{r}{2a} = q_1$

up to the first approximation,

we have, $q_1^2 - Q = \left(\dfrac{r}{2a}\right)^2 = \epsilon_1$, say

Then ϵ_1 will denote the first error. Therefore,

$$s_1 - s = \frac{\epsilon_1}{8d}$$

Similarly, for the second approximation, the error will be[1]

$$\epsilon_2 = \left[\frac{(r/2a)^2}{2(a+\frac{r}{2a})}\right]$$

and $s_2 - s = \dfrac{\epsilon_1}{8d}$

We shall now refer to a specific instance, in which[1]

$a = 1, \quad d = 1, \quad s = 60.$

---

(1) Folio 56, verso and 64, recto. Portions of the detail workings are not preserved in the existing manuscript. But they can be easily restored. Folio 56, verso and recto as preserved are,

| 880 84 | 964 168 | गुणित जातं | 848320 14112 | चत्वारिंश पृथक् स्थानं वर्ग .

| 1600 ! एष उपरापात्यशेषं | 846720 14112 | वर्त्यंजातं | 60 |

…… | 21 20 21 | तेषां वर्ग: तस्यात्

अकृते श्लिष्ठ कृत्यूनात् शेषच्छेदो द्वि संगुणम् ।
तद् वर्ग दल सश्लिष्ठ: हृति शुद्धि कृति क्षय: ॥
शेषच्छेदो द्विसंगुण कृ···

(carried over to next page)

The detailed workings given are

$$8ds=480,\ 2a-d=2.1-1=1.480+1=481$$

$$\sqrt{481}=21\frac{40}{42}=\frac{882+40}{42}=\frac{922}{42}$$

Then, $t_1=\frac{1}{2}\left[\frac{922}{42}-1\right]=\frac{880}{84}$

He ce, $s_1=\frac{t_1(t_1+1)}{2}=\frac{880}{84}\times\frac{964}{168}=\frac{848,320}{14,112}=60.$

And $\frac{\epsilon_1}{8d}=\frac{1}{8}\left(\frac{40}{42}\right)^2=\frac{1600}{14112}$

$\because\ s=s_1-\frac{\epsilon_1}{8d}=\frac{848320}{14112}-\frac{1600}{14212}=\frac{846720}{14112}=60$

Again for the second approximation,

$$\sqrt{481}=21\frac{20}{21}-\frac{(20/21)^2}{2(21+\frac{20}{21})}=\frac{425,042-400}{19,362}$$

$$=\frac{424,642}{19,362}$$

21		21 भ	
20	400 दल 1 संश्लिष्ठ	20	
21	441	21 +	

शेषंपात्य···द्वा भ।वित अधमुपरे उपरं गुणितव्यं वर्ग यावमजंये

……	425042	400	शेषं	424642
	19362	19362		19362

(Folio 56, recto and verso)

Folio, 64, recto, is—

405280	444004	—अर्धं कर्त्तव्यं—
38724	38724	

405280	444004	सगुण्य जातं । अ—ह्ररा ह्रेषु गुण
38724	77448	

…	179945941120	अस्य ऊर्ध्वं	160000+
	2999096352		

—Folio 64, recto.

$$\because\ t_2=\frac{1}{2}\left(\frac{424,642}{19,362}-1\right)=\frac{405,280}{38,724}.$$

Hence, $$s_2=\frac{t_2(t_2+1)}{2}=\frac{405,280}{38,724}\times\frac{444,004}{77,448}$$

$$=\frac{179,945,941,120}{2,999,096,352}$$

$$\frac{\epsilon_2}{8d}=\frac{40^4}{8\times 21^4\times(21\frac{20}{21})^2}=\frac{160,000}{2,999,096,352}$$

$$\therefore\quad s=s_2-\frac{\epsilon_2}{8d}=\frac{179,945,941,120-160,000}{2,999,096,352}$$

$$=\frac{179,945,781,120}{2,999,096,352}=60$$

Negative Sign

In the Bakhshali manuscript, a negative quantity is denoted by a cross (+) placed after the number affected. Thus, 11 7+ means 11 − 7. This is very remarkable. For in the manuscripts of Prthudakasvami (600 A.D.) and later Indian writers a dot is usually placed above the quantity for the same purpose, so that according to them 11—7 is denoted by 11 7°. The origin of the use of a cross for the negative sign has been the subject of much conjecture. Thibaut has suggested its probable connexion with the Diophantine negative sign ⋔ (reversed ψ, abbreviation for λεωψις, meaning "wanting",[1] This has been accepted by Kaye[2]. But in the opinion of Datta such a conjecture seems to be hardly reliable. For firstly, the Greek sign or minus is not ψ but an arrow-head (↑) and "it is now certain", observes Heath, 'that the sign has nothing to do with ψ.'[1] And arrow-head and a cross are too much different to be connected together, or too distinct to be confused for each other. Secondly, the Greek symbol itself is of doubtful origin. And above all, we are not sure if it is as old as it wili have to be for being the precursor of the Bakhshali cross. For there is no manuscript of the *Arithmetica* of

(1) *Indian Antiquary*, XVII, p. 34

(2) *Bakh. Ms.* 127, JASB, VIII (1912), p. 357.

(3) Heath, *Greek Mathematics*, Vol. II, p. 459.

Diophantus which is older than the Madrid copy of the thirteenth century A. D. and again in many cases in this work, the negative quantity is indicated by writing the full Greek word for "wanting" in its different case ending. So we cannot be sure if Diophantus did actually use that symbol for the minus sign[2]. Under such circumstances it will not be proper to assume the possibility of Greek connection for the negetive symbol of the Bakhshali manuscript, Hoernle thinks it—though he is not quite confident in this respect—to be the abbreviation ka of the word kanita or *nu* (नू or *una*, ऊन) of the word *nyuna*, (न्यून), both of which means "diminished" and both of which abbreviations, in the Brahmi characters, would be denoted by a cross[3]. Datta thinks that this supposition has got a very notable point in its favour. In the Bakhshali manuscript all other arithmetical operations are generally indicated by the abbreviations (initial syllables of the words of that import, though often the words are written in full and occasionally nothing is indicated at all).[1] So it will be very natural to search for the origin of its negative sign in that direction. In this way, Hoernle's hypothesis appears to be a very probable one. But its principal drawback is that neither the word *kanita* (कनित) nor the word *nyuna* (न्यून) is found to have been used in the Bakhshali work in connection with the subtractive operation. The nearest approach to that sign is that of *ksha*, क्ष, abbreviated from *kshaya* (क्षय, "decrease") which has been used several times, indeed more than any other word indicative of subtraction. The sign for ksha, (क्ष), whether in the Brahmi characters or in the Bakhshali characters, differs from the simple cross (+) only in having a little flourish at the lower end of the vertical line. The flourish might have besn dropped subsequently for convenient simplification.

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(1)' D. E. Smith, *History of Mathematics*, Vol. II, p. 396.

(2) D. E. Smith, *History of Mathematics*, Vol. II, p. 396.

(3) *Indian Antiquary*, XVII, p. 34.

(4) It may be noted that abbreviations of all sorts of things, mathematical as well as non-mathematical, have been freely used in the Bakhshali work (vide 62 of Kaye)

अ, a° (for *asva*, अश्व, horse.

ह, ha° (for *hasti*, हस्ति, elephant)

ऊ, u° (for *ustra*, ऊष्ट्र, camel)

(carried over to next page)

Least Common Multiple

The plan of reducing fractions to the lowest common denominator before adding or subtracting is known correctly to the author of of the Bakhshali mathematics, We have a few instances of its application in the work. In one instance[1], it is required to find the sum of the fractions.

$\frac{2}{1}$, $1\frac{1}{2}$, $1\frac{1}{3}$, $1\frac{1}{4}$, $1\frac{1}{5}$

They are first reduced to a common denominator (सदृशं क्रियते) so as to become,

$\frac{120}{60}$, $\frac{90}{60}$ $\frac{80}{60}$, $\frac{75}{60}$, $\frac{72}{60}$,

respectively. Finally, the sum is stated to be $\frac{437}{60}$.

गो, go° (for *go*, गो, cow)
दी, di (for *dinar*, दीनार, a coin)
लि, li (for *lipta*, लिप्त, a minute of an arc)
वि, vi (for *vilipta*, विलिप्त, a second of an arc)
*, शून्य, or zero or empty space, a cypher
[1] प, y, pa° (for *pada*, पद, a term)
and similarly of plants. In mathematical operations, भा (bha) for भाग, a divisor; शे (she) for शेषं, a remainder, मू (mu) for मूल a root, and फ (pha) for फलं, an answer.

(1) अर्ध त्रिभाग पादांशं पञ्चभाग षडंश च
…ततो प्रोक्षय: सदृशं क्रियते… जाता
| 120/60 | 90/60 | 80/60 | 75/60 | 72/60 | तत्र प्रोक्षय जातं 120 | 90 | 80 | 75 | 72
एषां योग कृते जात 437 अतो…शेषं 377 एष मणि मूल्यम् ।
Folio 1, recto and verso.

In a different instance[1], it became necessary to add up,

$\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}$

It is stated that the sum, after having reduced to a common denominator (हरस्मये कृते युतं), will be $\frac{163}{60}$. On reducing the fractions.

$\frac{12}{19}, \frac{4}{7}, \frac{6}{11},$

to a common denominator, they are stated to be respectively[2],

$\frac{924}{1463}, \frac{836}{1463}, \frac{798}{1463}.$

---

(1) करणं क्षय ।। संगुण्य कनका एष स्थापयते ।

$\left| \frac{1}{2} \right| \frac{2}{3} \left| \frac{3}{4} \right| \frac{4}{5} \right|$

तद् युतिर्भाजयेत् ततः हरसास्ये कृतं युतं $\left| \begin{matrix} \\ 163 \\ 60 \end{matrix} \right|$ संयुतैः···

Folio 17, recto,

(2) उदा० ।। अन्योन्य विदित विभवं······वणिक् द्वयम् ।

त्रि··· ··· ···दलं तथा

$\left| \begin{matrix} 7+ \\ 12 \\ 12 \end{matrix} \right| \begin{matrix} 3+ \\ 12 \\ 12 \end{matrix} \left| \begin{matrix} 5+ \\ 6 \\ 6 \end{matrix} \right|$

अंशां विशोध्य विशोधयेत् ऋणं स्थितम् । एष···क्रियते

$\left| \frac{19}{12} \right| \frac{7}{4} \left| \frac{11}{6} \right|$ कुर्यात्तत परिवर्त्तनम् $\left| \frac{12}{19} \right| \frac{4}{7} \left| \frac{6}{11} \right|$ च्छेदे

···जातमस्य $\left| \frac{924}{1463} \right| \frac{836}{1463} \left| \frac{798}{1463} \right|$

| 924 | 836 | 798 | एषां युतिं क्रियते···जाता | 2558 | च्छेद

प्रोझ्यं 1095 एतन्मणिमूल्यम्

Folio 2, verso.

A fairly difficult case is to simplify[2],

$$\frac{13\frac{1}{3}}{3\frac{1}{8}}+\frac{13\frac{1}{4}}{8\frac{1}{2}}+\frac{1\frac{1}{3}}{3\frac{1}{8}}+\frac{\frac{1}{2}}{1\frac{1}{2}}+\frac{1}{5\frac{1}{3}}+\frac{2\frac{1}{4}}{5}+\frac{12\frac{1}{2}}{33\frac{1}{3}}$$

The result is correctly obtained as $\frac{1807}{240}$

---

(२)·········आरयेत्

···सार्धं द्वादशामेवात्र भोजने मद्यमुत्तमेत्
सैंत्रिभाग त्रयस्त्रिंशै दिनैद् वाणिज्य कस्य तु ।
भाण्डारे द्वादशशत व जाराणां स्थितस्य वै ।
एष व्यय समुत्पत्ती क X कालं ब्रूहि पण्डित
करणविधानेन द्वादशशतस्य भाण्डारेंऽस्थितता

10	2	भा	13	3	भा	13	8	भा	1	3	भा	1	1	भा	1	5
1	1		1	1		1	1		1	1		2	1		1	1
2	3		3	8		4+	2		3	5			2			3

भा	2		1]	भा 12	33	भा	1	1	भाण्डा	1200	गुणितानि
	[1	1	1	1	1		1	360		1	
	[4	4	4	2	3						

Translation ;

······and also twelve and a half in thirty three, and one third days for the best wine for the consumption of merchants. In the treasure house was stored twelve hundred. Say O Pandita, how long can this expenditure continue.

The statement means—

$$\text{daily income} = \frac{10\frac{1}{2}}{2\frac{1}{3}} = \frac{9}{2}$$

$$\text{Daily expenditure} = \frac{13\frac{1}{3}}{3\frac{1}{8}}+\frac{13\frac{1}{4}}{8\frac{1}{2}}+\frac{1\frac{1}{3}}{3\frac{1}{8}}+\frac{\frac{1}{2}}{1\frac{1}{2}}$$
$$+\frac{1}{5\frac{1}{3}}+\frac{2\frac{1}{4}}{5}+\frac{12\frac{1}{2}}{33\frac{1}{3}}$$
$$=\frac{1807}{240}$$

(The daily loss is, therefore, $\frac{1807}{240}-\frac{9}{2}=\frac{727}{240}$

and $\frac{1200}{727+240}\times\frac{1}{360}=\frac{800}{727}$ is the period)

The method of finding the least common multiple is found in the *Ganita-Sara-Samgraha* of Mahavira (c. 850 A. D.[1]) and probably also in Prithudakasvami's commentary on the *Brahma-Sphuta-Siddhanta*[2], but not in works of Aryabhata, Brahmagupta and Bhaskara.

Arithmetical Notation—Word Numerals

The arithmetical notation generally empolyed throughout the Bakhshali work is the decimal place-value notation. This fact has been differently utilised by different writers. On the one hand, Hoernle[3] and Buhler[4], who believe in the antiquity of the Bakhshali mathematics, consider it as evidence of the earlier date of the discovery of that notation by the Indians. On the contrary, Kaye[5] who believes in the non-Indian origin of the place-value notation and in its late introduction into India, considers its general adoption in theBakhshali work as proof against the hypothesis of the previous writers about the early date of this work. It is now definitely known that Kaye's notions about the origin of the place-value notation is wrong. It was invented in India about the beginning of the Christian era, probably a few centuries earlier. For this, one may consult the writings of Datta[6]. But apart from that controversy it should be noted that the nearly exclusive application of the notation in the Bakhshali work is very much noteworthy in asmuch in almost all the available Indian mathematical treatises, save and except the *Arya-Bhatiya* of Aryabhata (499 A. D.), we find copious use of the word-numerals. There is, however, evidence to show that the author of the Bakhshali work did know the principle of the word numeral system of arithmetical notation. In it we find the use of the word

(1) GSS. *Ganita Sara-Samgraha*, III, 56.

(2) Colebrooke, *Hindu Algebra*, p. 281, footnote; p. 289 fn. Also pp. 178—9.

(3) *Indian Antiquary*, XVII, p. 38.

(4) *Indian Paleography*, p. 82,

(5) *Bakh. Ms.*, Sec. 131.

(6) Datta, "*A Note on the Hindu-Arabic Numerals*" (*Amer. Math. Month.*, Vol. 33, 1926, pp. 220—1), "Early literary evidence on the use of the zero in India" (*Ibid.*, pp, 449—54); "the present mode of expressing numbers" (*Ind. History Quart.*, Vol. 3, 1927, pp. 530—40 "*Al Biruni and the origin ofthe Arabic Numerals*" (*Proc. Benares Math. Soc.*, Vol. 7, 1928)·

रूप, rupa (=1), रस, rasa (=6,[1] and पाद, pada (=¼)[2] with numerical significance. The use of the last word is as old as the Vedas. The first occurs as early as in the *Jyotisa Vedanga*[3] (c. 1200 B.C.) and second in the *Chandah-sutra* of Pingala (before 200 B. C.)[4]. Again, in speaking of a very large number,

2653296226447064994...83218

the Bakhshali mathematics writes[5] :

Sadvimsasca tripancasa ekonatrimsa eva ca

Dvasa (sti) sadvimsa catuhcatvalimsa saptati

Catuhsastina (va)... msanamtaram

Trirasiti ekavimsa asta....... pakam,

Clearly the principle of the word-numeral system has been followed in this instance. The only departure from its popular features lies in (1) the use of the number names in the place of the word-names and (2) the adoption of the left-to-right system in the arrangement of the figures. But, according to Datta[6], these features,

---

(1) त्रिदिने आर्जयेपञ्चभूतकोमेक पण्डितः ।
द्वितीयं पञ्चदिवसे रसमार्जयते बुधः ।।

—Folio 60, verso.

(2) ···विभक्त | 1/4 | पूर्वंगत ३५ एष पादेर्गुणितं | 35/4 |

—Folio 4, recto.

(3) Yajus-Jyautisa, 23: Arsa-Jyautisa, 31.

(4) Chandah—Sutra, VI, 34: VIII, 2, 3.

(5) उदा० षड्विंशश्च त्रिपञ्चाशैकोन त्रिंशैव च ।
द्वाश ...षड्विंश चतुश्चत्वालिंश सप्तति ।
चतुष्षष्ठि नव न्शनं तरम् ।
त्रिराशीत्येकविंशाष्ट···पकम् ।।
(2653) 296226447064994...(83218)

—Folio 58, recto.

(6) See Datta's papers in the *Bangiya Sahitya Patrika*, 1335 B. S. (1928), pp 8-30 on the comprehensive history of the origin and development of the word-numerals,

though not common, are not altogether foreign to the system. Once at least the author has followed the right-to-left sequence. For the compound word 'Catuh panca' (चतु्..पञ्च) has been used to denote once | 4 | 5 | and again | 5 | 4 |. (Folio 27, recto).

Regula falsi

The rule of "*false position*" or "*supposition*" is used in two ways in our text[1] in solving linear equations.

(i) $f(x)=p$ is solved by assuming a value e for x. This gives $f(e)=p'$ and the correct value of x is ep/p'.

An example is,

The amount received by the first is not known. The second receives twice as much as the first, the third thrice as much as the first two and the fourth four times as all the others. Altogether they receive 300. How much did the first receive ?

Suppose the first receives one, then second receives 2, the third 9, and the fourth 48 ; or altogether they receive 60. Actually therefore, the first received $300/60=5$.

(ii) If $f(x)=p$, reduces to $bx+c=p$ and $f(e)=p'$ reduces to $be+c=p'$, then $x=(p=p')/(b+e)$, which appears to have been considered useful when it was desired to keep b and c unchanged.

For example, if $x_1+x_2=a_1$, $x_2+x_3=a_2$, $x_3+x_1=a_3$, we have $2x_1+(a_2-a_1)=a_3$; and if in place of x we put e then $2e+(a_2-a_1)=a'_3$ and the correct value of x_1 is $(a_3-a_3')/(2+e)$.

This rule of 'false position' is interesting as being, in a way, a precursor of algebraic symbolism. It connotes the idea of an un-

(1) No rule is preserved in the text but Bhaskara II gives the following (*Lilavati*, 50) :

"Any number assumed at pleasure is treated as specific in the particular question, being multiplied and divided, raised and diminished by fractions, then the given quantity, being multipled by the assumed number and divided by the result yields the number sought.

known quantity and even of a symbol for that quantity (e. g., as in our text) but it does not embrace the notion of such a quantity being subject to operation or being isolated. As soon as we introduce algebraic symbols, the rule, as it were, disappears. Algebraically, the rule solves $bx=p$ by $x=ep/p'$, where $p'=be$, which shows that the transformation from $bx=p$ to $x=p/b$ was not conceived.

There is no algebraic symbolism in our text but it may be noted that Bhaskara gives both the *regula falsi* and also an early form of algebra'c symbolism. Neither Aryabhata, Brahmagupta nor Sridhara gives this rule or makes use of the principle. On this, Kaye says that its occurrence in the *Lilavati*, therefore seems to indicate that it was introduced into northern India after the time of Sridhara (XIth cent.). Mahavira (IXth cent.), however, uses the method in rather a special way in connection with a geometrical problem[1].

But such a surmise according to Datta is wholly baseless. We shall sum up here the views advanced by Datta :

The rule of '*false position*' has been applied in certain cases in the *Ganita-Sara-Samgraha* of Mahavira. For instance, for finding out unknown quantity (*avyakta*, अव्यक्त; *ajnata* अज्ञात), the sum of the various fractional parts of which is known, Mahavira says :

"The given sum, when divided by whatever happens to be the sum arrived at in accordance with the rule (mentioned) before by putting down one in the place of the unknown (element in the compound fractions), gives rise to the (required) unknown (elements) in (the summ'ng up of) compound fractions[2]".

We have a few other instances of this kind in the work[3]. Further Mahavira has applied the method of supposition in solving certain geometric problems[4]. Hence Kaye is not truely accurate when

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(1) *Ganita-Sara-Samgraha.*, VII. 112.

[2] *Ganita-Sara-Samgraha* III. 107. Compare the original expression in the work रूपंन्यासव्य etc. with the passage शून्य स्थाने रूपं दत्वा and similar other passages in the Bakhshali work [folios 25, verso and 26, recto; compare also folios 23, 23]

[3] *Ibid* III, 122, 132, 135--7

[4] *Ibid.* VII, 112, 221. This should be more accurately called the geometric prototype of the *regula falsi* of algebra.

he says : "Mahavira (9th century), however, uses the method in rather a special way in connection with a geometrical problem"[1]. In fact Mahavira has made more extensive use of the method in connection with certain algebraical as well as geometrical problems. Stlll it is quite true that he has not made as general use of the method as is found in the Bakhshali work, or even in the *Lilavati*[2]. In any case, Kaye's hypothesis of the foreign import of the rule of false position into India after the eleventh century must be abandoned. It should be further noted that Mahavira (c. 850) is a contemporary of the earliest Arab algebraist to use that rule, namely, Al khowarrizmi (c. 825). Hence it is quite certain that the Hindus have not taken the *regula falsi* from the Arab scholars, f they have done so at all from a foreign nation[3].

Known still earlier in India—

It should be observed that the rule of false position was reported to by the Arab and European algebraists at the early stage of development of their science when there were no symbols. It almost disappeared from amongst them, as it is bound to do so, with the introdnction of a system of notations[1]. It will be nothing unreasonable to expect that such had been the case with that rule in India too, if it was ever followed here. Now it is a well established fact that the Hindus reached Nessel-mann's third and the last stage of development of the science of algebra long before all the other nations of the world. They invented a good system of notations

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(1) *Bakh. Ms.*, Sec. 72, This statement of Kaye followed by another of the same kind, "It (the regula falsi] occurs in no Indian work until the time of Mahavira" Sec. 134], will obviously contradict his previous statement, "Its occurrence in the *Lilavati*, therefore, seems to indicate that it was introduced intonorthern in India after the time of Sridhara [11th century]" Sec. 72]. Thus it appear that Kaye is not sure of his own grounds.

(2) Certain problems in the Bakhshali work, the *Lilavati* [pp. 10 et. seq], the *Trishatika* [pp. 13 et seq] and *GSS* [IV, 5—32] which are of the same kind but differ only in detail, have been solved in the first two works expressly by the *regula falsi*, but not so in the other two works, thought in them the unknown quantity has been tacitly assumed to be one.

(3) It should be noted in this connection that while there are ample proofs in the writings of the early Arab scholars of their heavy indebtedness to Indian Mathematics it is still to be proved that the Indians took anything in return from them.

by the beginning of the seventh century of the Christian era. It has been laid down by Brahmagupta (628 A D) that a thorough knowledge of algebraic symbols (*varna*) is an essential qualification for a good algebraist[2]. We find mention of a symbol for the unknown even in the *Aryabhatiya* of Aryabhata (499 A. D.). So the method of false position must have disappeared from India before that time. Or it is at least bound to have been regulated to a very inferior position from that time. This will account for the absence of the method from the works of Aryabhata and Brahmagupta as well as for its limited application in the *Ganita-Sara-Samgraha* of Mahavira. There is now left no direct evidence from the Hindu source to show that that method was followed in India before the fifth century A. D. Unfortunately no Hindu treatise on arithmetic or on algebra which can be definitely referred to that period has survived and come down to this day with the exception of the Bakhshali manuscript. There is, however, external evidence. A mediaeval Arabic writer of note, possibly Rabbi Ben Ezra (b. 1095) refers the origin of the rule of false position to India[1]. And if our hypothesis about the origin of the term *yavat tavat* (यावत् तावत्) for the unknown in Hindu algebra be true, it is in all probability so, then there will remain very little to doubt that the rule was known in India much earlier.

The rupona method

There are several references to the *rupona* (रूपोणे) method, and the phrases *rupona karanena* (रूपोण करनेण) or *pratyaya rupona karanena* (प्रत्यय रूपोण करनेण) occur. In all the cases in arithmetical progression according to the rule.

$$s+[(t-1)\frac{d}{2}+a]t$$

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[1] Smith, D. E , *History of Mathematics*, Vol. II, p. 437.

]2] Brahmagupta says : 'By the pulverizer, cipher, negative and affirmative quantities, elimination of the middle term, colours [or symbols and factum, well understood, a man becomes a teacher among the learned and by the affected square' (Colebrook, *Hindu* Algebra, p. 325).

[1] Smith, D. E., *History of Mathematics*, II, p. 437, footnote

The recurrence of the phrase *rupona karanena* (रूपोण करणेन) seems to imply that the rule in question began with the term rupona which corresponds to the (t−1) of the formula[1]. The rule is not preserved in our text but we find the following in the *Ganita-Sara-Samgraha* of Mahavira[2].

"The number of terms is diminished by one, halved and multiplied by the increment. This when combined with the first term of the series and multiplied by the number of terms becomes the sum of all".

The rule is exemplified in two ways in our text of which the following are particular cases :

(i) आ | 1
1 | उ० 1 | प० 19 |

रूपोन करणेन फलम् | 19
1

---

(२) स्थाप | आ 1
1 | उ० 1
1 | प० 19
1 | रूपोन करणेन फलम् 19
1 |

Folio 9, recto, verso.

आ० 3 | उ० 4
1 | प० 3
1 | रूपोन करणेन फलम् रू० 21

Folio 7, verso.

पदं ईना उभये स्थापितव्या रूपोणाकरणे फलम् | 21 | द्वि
किं प्रभूतेपि लिखिते ।। | 217 |

Folio 4, verso

प्रत्ययं । रूपोणाकरनेण फलम् | 65
द्वि 65 | एषपदम् ......

Folio, 5 recto.

(1) Is this a faint echo of Pythogorean style ? Anatolius writes : The Pythagoreans state that their master, in connectiun with numbers that form a right-angled triangle, showed how such could be composed by means of unity'. i. e.,

$$\left[\tfrac{1}{2}(a^2-1)\right]^2+a^2=\left[\tfrac{1}{2}(a^2+1)\right]^2$$

See P. Tannery : '*Memoires Scientifiques, III,* 14.

(२) रूपेनोनो गच्छो दलीकृतः प्र चयतादि तो मिश्रः ।
प्रभवेन पदाभ्यस्तस्संकलितं भवति सर्वेषाम् ।।

GSS. II, 63.

which means : $a=1$, $d=1$, $t=19$; by the rupona method, the answer is $s=1$ 0.

(ii) In section 'C' the application of the rupona method is always worked out step by step. For example, when $a=5$, $d=3$, $t=178/29$; then $s=[(t-1)\frac{3}{2}+5]\,t$; and the rupona method is applied as follows[1] :—

$$(t-1)=\frac{149}{29}\,;\quad 3\times\frac{149}{29}=\frac{447}{29}\,;\quad \frac{447}{29}\times\frac{1}{2}=\frac{447}{58}\,;$$

$$\frac{447}{58}+5=\frac{737}{58}\,;\quad \frac{737}{58}\times\frac{178}{29}=\frac{65593}{841}$$

$$=77\frac{836}{841}$$

The origin of that name is supposed by Hoernle[2] and Kaye[3] to be lying in the fact that "the rule in question began with the term rupona [(रूप+ऊन), i.e. one less] which corresponds to the $(t-1)$ of the formula". The term rupona literally means, "deducting one". As the rule is not preserved in the available portion of the Bakhshali mathematics, it is not possible to verify this

(1) | 6 1 8 60* 16च 60* 33लि 60* 6 वि 60* शे• 6 29 |

$\frac{447}{29}$ दलित | $\frac{447}{58}$ | ···सास्येयुतं | $\frac{737}{58}$ | पद

ध्ना । तत्र पदं | $\frac{178}{29}$ | अनेन गुणितं जातं | $\frac{65593}{841}$

श्लि···त्य शेषं क्रियते । | $\frac{65569}{841}$ | भागे हृते

प्रत्ययं त्रैराशिकेन | $\frac{1}{1}$ | $\frac{7}{1}$ यो० | $\frac{178}{9}$ फलम्

योजन 42 शे $\frac{28}{29}$ | नियंततेन ···77

(Folio 7, recto.)

(2) *Indian Antiquary*, XVII, p. 47.

(3) *Bakh. Ms.*, Sec. 73

supposition. Kaye, however, points out that the rule has very nearly the same beginning in the *Ganita-sara-samgraha*[1] of Mahavira : *Ruponena gaccho dali krtah...* (रूपोनेन गच्छो दलो कृतः). According to Datta, the above interpretation of the origin of the term rupona karana, though not impossible, does not appear to be very convincing. The technical terms which are commonly used in the Bakhshali mathematics in connection with the arithmetical progression, such as *adi, prabhava, caya, uttarapada dhana*, etc., are all the same as in the other ancient Indian treatises; the name *rupana karana* is unique for it. It is not met with elsewhere. It is further noteworthy that no other term in the Bakhshali mathematics, or in any other Indian mathematical treatise, is known to have been formed in the same way, with the opening word rule.

It may be noted here that in the Bakhshali mathematics, the word *rupa* (रूप) occurs also with different significance than unity. For instance, we find[2]

“......(bha) jitam $\left|\begin{matrix}4\\2\end{matrix}\right|$ jātam | 2 | labdham sarupa | esa

rupadhikam | 3 | esa kala... a° $\left|\begin{matrix}3\\1\end{matrix}\right|$ u° $\begin{matrix}4\\1\end{matrix}\Big|$ pa° $\begin{matrix}3\\1\end{matrix}\Big|$

rupona·karanena phalam ru° 21 | dvitiyasya trairasikena...

... $\left|\begin{matrix}1\\1\end{matrix}\right.$ di° $\left|\begin{matrix}7\\1\end{matrix}\right|\begin{matrix}3\\1\end{matrix}\Big|$ di° pha° ru° 21” ||

or ‘ ...divided becomes 2; ‘quotient plus 1 (rupa),’ this increased by 1 bec mes 3; which times...by the ruponakarana, the result is ru° 21. Of the second, by the rule of three, the result is ru° 21”.

(1) ii, 63.

(2) Folio 7, verso. The problem is

$$7t = \left[(t-1)\cdot\frac{4}{2} + 3\right] t, \text{ whence } t = \frac{2(7-3)}{4} + 1 = 3.$$

By the rupona method, $s = [(3-1)\,\frac{4}{2} + 3]\,t = 21$ and by the ‘rule of three’ 1:7::3:21.

In this passage the number 21 has twice been marked as ru[2], abbreviated from rupa. Again in a *sutra* (folio 8, recto) related to an arithmatical progression, we find the passage *labdham rupam yinirdiset*, (लब्धं रूपं विनिर्दिशेत्), that is, 'the quotient should be indicated as *rupa*'. Here again the term *rupa* seems to have a purely technical significance. There are other instances in which *rupa* does not mean one, but is used in connexion with an integer[1]. Similar use of the word *rupa* is found in later Indian mathematical treatise where it denotes, besides 1, an integer or the integral part of a mixed fraction[2]. I venture to amend the word *rupona* (रूपोन) to *rupana* (रूपण)[3] Then it will mean 'making rupa' which means 'known or absolute number,' 'known quantity as having specific form[4]'. So rupana-karana (रूपणकरण) will mean the method of making absolute number', that is, 'totalization' or 'summation'. This hypothesis will be strongly supported by the expression 'rupana karanena phalam rupam rupa 21' (रूपण करणेन फलं रूपं रूप 21, or 'by the method of making *rupa* the result is rupa 21').

Proofs

To many solutions are attached proofs, which are generally introduced by the term *pratyayam*, प्रत्ययम्. 'proof' or 'verification'. This is sometimes amplified into *pratyaya-trairasikena* (प्रत्यय त्रैराशि-

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(1) Folios 21, recto; 60, recto; etc.

त्रिभाग ··· दिने तथा । त्रिरूप पञ्चभिर्दिनैर्पां द ··

—Folio 21, recto.

एकोनविंशतिम | गावो 10/1 | रूप 8/1 | ··· | ··· विवरितास्ति

—Folio 60, recto.

(2) See *Br.Sp. Si*, XII, 2; *Trisatika*, pp. 7 et seq; *Lilavati*, pp. 6, 7; *Bija*., pp. 2 *et seq*; (Colebrooke : *Hindu Algebra*, p. 149).

( ) Rupona (रूपोण = रूपोन) may be an archaic form of rupana (रूपण)

(4) See Monier-Williams, *Sanskrit English Dictionary*, revised by Cappaller and Leumann, on *rupa*. Compare this use of the word *rupa* with its use in algebra in the sense of absolute known number in an equation.

केन) 'proof by the rule of three', or *pratyaya-rupona-karanena* प्रत्यय रूपोनकरणेन 'proof by the rupona method'.

Sometimes the 'proof' seems to be merely a matter of rewriting a statement in another form, but generally the answer is utilized and one of the original terms of the problem is rediscovered. Occasionally the 'proof' consists of a solution of the original problem in another way, e. g., by steps; but sometimes it deals with a subsidiary aspect of the original problem. In certain problems approximations are employed and then the 'proof' may be described as a process of reconciliation. These 'reconciliations' entain a comparatively high degree of mathematical skill. Occasionally several different proofs are attached to a problem. The following are specimens.

(1) Problem $\frac{5}{3}t-7=\frac{6}{5}t+7$

Solution $t=2\times7/(\frac{5}{3}-\frac{6}{5})=30.$

Proof [1]—3:5 : : 30 : 50, 5:6 : : 30:36 and 50−7=36+7

—Folio 61, recto.

(2) Problem—A gives $2\frac{1}{2}$ in $1\frac{1}{2}$ days,

B gives $3\frac{1}{3}$ in $1\frac{1}{3}$ days.

C gives $4\frac{1}{2}$ in $1\frac{1}{4}$ days.

In what time they have given 500 dinaras altogether ?[2]

---

(1)... अनेन कालेन समधना भवन्ति ॥
प्रत्ययं त्रैराशिकेन क्रियते

3/1	5/1	30/1	फ० 50

प्रथमे द्वितीयस्यस्सप्त दत्ता | 7 |

शेषं 43 | ....43 |

5/1	6/1	30	36

43 | एते समधना जाता

—Folio 61, recto.

(2)... | 473500/947 | वर्तित जाता फलं दी 500

(carried over to next page)

Solution, $t = \dfrac{500}{\dfrac{2\frac{1}{2}}{1\frac{1}{2}} + \dfrac{3\frac{1}{2}}{1\frac{1}{3}} + \dfrac{4\frac{1}{2}}{1\frac{1}{4}}} = \dfrac{60,000}{947}$ days

And A gives 100,000/947, B gives 157,000/947, C gives $\dfrac{216,000}{947}$ dinaras.

Proof. $2\frac{1}{2} : 1\frac{1}{2} : : \dfrac{100,000}{947} : \dfrac{60,000}{947}$

$3\frac{1}{2} : 1\frac{1}{3} : : \dfrac{157,000}{947} : \dfrac{60,000}{947}$

$4\frac{1}{2} : 1\frac{1}{4} : : \dfrac{216,000}{947} : \dfrac{60,000}{947}$

Folio 22, recto.

(3) Problem. One earns e and spends f daily. How long will a capital of C last ?

Solution, $= t \dfrac{C}{(f - e)}$

Proof. $1 : f : : t : F$ (the total expenditure).

$1 : e : : t : E$ (the total earnings) and $F - E = C$.

(4) Problem, If 7 are bought for 2 and 6 sold for 3, and the capital is 24, what will be the profit ?

Solution. $p = \dfrac{C}{c/s - 1} = 24\left(\frac{7}{2} \div \frac{6}{3} - 1\right) = 18$

Proof. 2 : 7 : : 24 : 84 (the number of articles)

and 6 : 3 : : 84 : 42 (the total proceeds),

and 42 − 24 = 18.

or ...$1 : c : : C : n,\ s : 1 : : n : C + p$

and $C + p - C = p$,

2 दी० 1 2	1 दी० 1 2	100000 दी० 947	फलं दी० 60000 947
3 दी० 1 2	1 दी० 1 3	157000 दी० 947	फलं दी० 60000 947
4 दी० 1 2	1 दी० 1 4	216000 दी० 947	फलं दी० 60000 947

Folio 22, recto.

(5) **Problem.** $x(1-\frac{1}{2})(1-\frac{1}{4})(1-\frac{1}{5})=x-280$

Solution.[1] $\frac{400}{2}=200,\ 400-200=200$

$\frac{200}{4}=50,\ 200-50=150$

$\frac{150}{5}=30,\ 150-30=120$

and $400-120=280$.

—Folio 52, verso.

(6) **Problem.**

$(1+\frac{1}{2}(2+\frac{1}{2}(3+\frac{2}{3}(4+\frac{3}{5}(5+\frac{1}{2})))))$

Solution $=\frac{23}{16}$.

Proof. $(((((\frac{23}{16}-1)\,2-2)\,2-3)-4)\,\frac{2}{3}-5-\frac{1}{2})=0$.

(7) **Problem.** Solve $x+5=s^2,\ x-7=t^2$

Solution. $x=\left[\frac{1}{2}\left(\frac{5+7}{2}-2\right)\right]^2+7=11$

Proof. $11+5=4^2$; $11-7=2$.

—Folio 59, recto.

(8) **Problem.** $F=\dfrac{\frac{1}{2}(1)+\frac{1}{3}(2)+\frac{1}{4}(3)+\frac{1}{6}(4)}{1+2+3+4}$

Solution. $F=\dfrac{163\div 60}{10}=\dfrac{163}{600}$

Proof. $10:\frac{163}{60}::1::\frac{163}{600}$

(1) अस्य द्वयानां शतानां पाद···र्द्धम्
शतं भवति 150 अत्रापि पञ्चभाग ३० । एवं···

1	1	1	फ० पिण्डा 280
2	4	5	
	1	1	
	2+	4+	
		1+	
		2	

पञ्चमी जातकरणं कृत...280 / अंशयुति $\begin{array}{|c|}28\\40\end{array}$ भक्तं $\begin{array}{|c|}40\\28\end{array}$

धनु 280 गुणितं जातं 400 एष फलं भवति

—Folio 52, verso.

$$10: \frac{163}{60} :: 2 :: \frac{163}{300}$$

$$10: \frac{163}{60} :: 3 :: \frac{163}{200}$$

$$10: \frac{163}{60} :: 4 :: \frac{163}{150}$$

or, since $F = \frac{\Sigma f \times w}{\Sigma w}$, $\Sigma w : \Sigma f \times w :: w_1 : w_1 F$

(9) Problem. $Dt = \left[\frac{(t-1)d}{2} + a\right] t$

Solution. $t = \frac{2(D-a)}{d} + 1$

Proof. By the rupona method, $s = \left[(t-1)\frac{d}{2} + a\right] t$

and $Dt = s$.

(10) Problem. $s = \left[\frac{(t-1)d}{2} + a\right] t$

Solution. $t = \{\sqrt{(2a-d)^2 + 8s} - (2a-d)\} \div 2d$

—Folio 65, verso.

Generally the solution is an approximation t_1 or t_2 depending on the method of evaluation of the surd quantity; and the rupona method gives s_1 or s_2, neither of which is the same as the original s. A process of reconciliation is therefore called for, and this is given by $s - s' = \frac{e}{8d}$ where s' is the approximation to s given by the proof and e may be termed the square-root error. Problem. $a = 1$, $d = 1$, $s = 60$. First solution, $t_1 = \{\sqrt{(2.1-1)^2 + 480} - 1\} \div 2.1$

$$(21\frac{40}{42} - 1) \div 2 = \frac{880}{84}$$

—Folio 65, verso.

First proof. $s_1 = t_1 (t_1 + 1) \div 2 = \frac{880}{84} \times \frac{964}{168} = \frac{848,320}{14112}$

$$e_1 = \left(\frac{40}{42}\right)^2 \text{ and } s = s_1 - \frac{e_1}{8d} = \frac{848,320 - 1600}{14112} = 60$$

—Folio 56, verso.

Second Solution. $t_2 = \left(\frac{424{,}64?}{19362} - 1\right) \div 2 = \frac{405{,}280}{38724}$.

Second Proof. $s_2 = \frac{405{,}280}{38724} \times \frac{444{,}004}{77448} = \frac{179{,}945{,}941{,}120}{2{,}999{,}096{,}352}$.

$$\text{Now}\quad e_2 = \left(\frac{40}{42}\right)^2 \div 4\ \left(21 + \frac{40}{42}\right)^2$$

$$\text{and}\quad \frac{e_2}{8d} = \frac{160{,}000}{2{,}999{,}096{,}352};$$

$$\text{and}\quad s = s_2 - \frac{e_2}{8d} = \frac{179{,}945{,}781{,}120}{2{,}999{,}096{,}352} = 60$$

—Folio 64, recto.

(11) The problems in section 'G' are characterised by numerous proofs—as many as five separate proofs being attached to one example. The following is fairly typical[1]

Problem. $x = 500(1-\frac{1}{2})\ (1-\frac{1}{4})\ (1-\frac{1}{4})\ (1-\frac{1}{4})$

Solution. $x = 158\frac{13}{64}$.

Proof (i). $x'\ (1-\frac{1}{4})\ (1-\frac{1}{4})\ (1-\frac{1}{4})\ (1-\frac{1}{4}) = 158\frac{13}{64}$.
whence $x' = 500$.

This is written horizontally.

Proof (ii). $x'\ (1-\frac{1}{4})\ (1-\frac{1}{4})\ (1-\frac{1}{4})\ (1-\frac{1}{4}) = 158\frac{13}{64}$, whence $x' = 500$. This is written vertically.

---

(1) उदा० चतुर्भाग मलदग्ध सुवर्ण शत–पञ्चकम् ।
... ... अथ प्रत्यय –

0 158 सु० फलं मूल 500 ॥ पुनरेव प्रस्तार कमंम्
1 1
1 + 5* तो०
4 1
1 + 64
4

500	1	1	1	1	फलं शेष
1	1 +	1	1	1	
+	4	4 +	4 +	4 +	तदिति

1 +
4
1

शेष 158 तो० 1 शे० 1
64

—Folio 11, verso.

Proof (iii), $x' = \dfrac{158\frac{13}{64}}{(1-\frac{1}{4})(1-\frac{1}{4})(1-\frac{1}{4})(1-\frac{1}{4})} = 500$

Proof (iv)—By steps,

$500 \div 4 = 125, \; 500 - 125 = 375;$

$375 \div 4 = 93\frac{3}{4}, \; 375 - 93\frac{3}{4} = 281\frac{1}{4};$

$281\frac{1}{4} \div 4 = 70\frac{5}{16}, \; 281\frac{1}{4} - 70\frac{5}{16} = 210\frac{15}{16};$

$210\frac{15}{16} \div 4 = 52\frac{47}{64}, \; 210\frac{15}{16} - 52\frac{47}{64} = 158\frac{13}{64}.$

—Folio 11, verso.

Solutions.

The solutions are sometimes very detailed, proceeding most carefully step by step, so that they become expositions in general terms of the processes involved. Also they often give actual quotations from the rules,-to such an extent sometimes that the original wording of the rule can be reconstructed. Unfortunately, however, such helpful suggestions are more often missing than not : in many cases the particular portions of the manuscripts are lacking or damaged beyond repair. In some sections, e. g. section 'M', nothing but the bare answer is generally given, and in others the outline only of the working is given.

The following examples illustrate these remarks :

(i) The problem is $DT+Dt=\{(t-1)+a\}\,t$ where a and d are respectively the first term and common difference of an arithmetical progression, and t the number of terms, to be found. The rule is

$t=\{2(D-a)+d+\sqrt{(2(D-a)+d)^2+8DdT}\}\div 2d$ and the particular example gives $D=5$, $T=6$, $a=3$, $d=4$. The actual working of the solution, translated as literally as possible, is as follows :

"The daily rate diminished by the first term;* the daily rate is five yojanas, 5; the first term is 3; their difference is 2: this doubled is 4: *this increased by the common difference is 8; and squared is 64 which is known as the *ksepa* quantity.* *Multiplied by eight* ; the fixed term (30) multiplied by eight is 240; and multiplied by the common difference—multiplied 960. *The quantity known as *ksepa* is added*. Now the quantity known as kshepa is 64, which added gives 1024. The root of this is 32; the quantity set aside is 8 and this added gives 40. *Divided by twice the common difference: *twice the common difference is 8—divided 5."

* The phrases marked off by asterisks are quotations from the *sutra* or rule.

The phrases placed between asterisks are quotations from the rule. A perfectly literal rendering would not be very intelligible to the ordinary reader and one or two gaps have been filled in; but the translation is a perfectly fair representation of the original.

(ii) The problem may be represented by $x(1-\frac{1}{3})\,(1-\frac{1}{4})\,(1-\frac{1}{5})= x-24$ and the solution is given as follows :—

Having calculated the loss on unity the terms become $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$, and these multiplied together give $\frac{2}{5}$. This subtracted from unity gives $\frac{3}{5}$, which, inverted and multiplied[1] by the given amount 24, is $\frac{5}{3}$ of $24=40$.

(iii) Something travels 3 *yavas* a day. How long will it take to go 5 yojanas ? Here the solution is indicated by the following proportion only :

$$3\ ya^{o} : \frac{1}{360}\text{years} : : 5\times 4{,}608{,}000 : 21{,}333 \text{ years } 4 \text{ months.}$$

Symbols for the Unknown

In the Bakhshali mathematics the unknown quantity is referred to by the symbol O, which is called 'sunya,' शून्य ('*void*' or '*empty*')[2]. Datta thinks that strictly speaking it is not a symbol for the

1. Literally "divided and multiplied" into. This is generally used to indicate division by a fraction.

2 (*i*) द्विगुणं द्वितीयस्य प्रथमा···तीय । प्रथमा चतुर्गुणं चैव चतुर्थे चैव दत्तवान् च न्शतमेकं द्वयानुगम् । वदस्व प्रथमे दत्तं किं प्रमाणम्

स्य··· | $\frac{0}{1}$ | $\frac{2}{1}$ | $\frac{3}{1}$ | $\frac{4}{1}$ | दृश्य $\frac{200}{1}$ | शून्यमेकयुतं कृत्वा

1 | 2 | 3 | 4 | ···क्षेप युक्त्या फलम्

| 20 | 40 | 60 | 80 | एवं 200 ॥ एषां

| आ $\frac{20}{1}$ | उ $\frac{20}{1}$ | प $\frac{4}{1}$ | रूपोणाकरणेन फलं 20

—Folio 22, verso.

The problem of this type :

"A certain amount was given to the first, twice that to the second, thrice it to the third and four times to the fourth. State the amount given to the first and the shares of others, if the total amount given was 200."

The shares are represented by 0, 1, 2, 3, 4. (Here zero stands as a symbol for the unknown). Now having added 1 to the unknown (nought), the sum is $1+2+3+4=10$, which shows the proper share of the first is 200/10 or 20, and the series is $20+40+60+80=200$.

The proof by the rupona method gives, (*Contd. on the next page*.

unknown as has been supposed by Hoernle[1] and Kaye[2]. For the same symbol has also been used for the 'zero' (sunya) for the decimal arithmetical notation. This is, indeed, its true significance. Its use in connexion with an algebraic equation, in a sense other than for arithmetical notation, is simply to indicate that the quantity which should be there is absent or not known[3]. Hence its place in the equation is left vacant and this is clearly indicated by putting the sign of emptiness there. Or in short, the use of the truely arithmetical symbol for zero in an algebraic equation is a clear proof of the want of a symbol for the unknown in the Bakhshali mathematics. Correctness of this interpretation will be borne out by the facts [1] that this symbol does nowhere enter into any operation, as it ought to have done had it been truely a symbol for the unknown, and [2] that often times it is referred to as sunya, sthana or the 'empty place', proving thereby that nothing is in that place[4]. This

---

$$\left[(4-1)\frac{20}{2}+20 \right] 4 = 200.$$

2(ii)...च द्विगुणं—

... प्रथमस्य तु किं भवेत्

$\left|\begin{matrix}0\\1\end{matrix}\right|$ तदा $\begin{matrix}2\\1\end{matrix}\Big|$ तदा /

यदृच्छा विन्यसे शून्ये...च्छा | 1 | तदावर्गं तु कारयेत्

$\left|\begin{matrix}\cdot\\1\end{matrix}\right|\begin{matrix}2\\1\end{matrix}\Big|\begin{matrix}2\\1\end{matrix}\Big|\begin{matrix}3\\1\end{matrix}\Big|\begin{matrix}6...\\1...\end{matrix}\Big|$ प्रक्षिपे गुणितं 1 | 2 | 6 | 24 | ...

...प्रक्षिप्तं 33 । दृश्यं विभजेत् $\left|\begin{matrix}132\\33\end{matrix}\right|$ वर्त्यंजातं $\left|\begin{matrix}4\\1\end{matrix}\right|$

Folio 23, recto.

The example may be represented by

$x+2T_1+3T_2+4T_3=132$, where T_1, T_2, etc, represent the values of the first, second, etc. terms. Make $x=1$, then the terms are $1+2+6+24 = 33$, whence $x=\frac{132}{33}=4$ and the series becomes, $4+8+24+96 = 132$ (Kaye).

(1) *Indian Antiquary*, xii, p. 90; xvii, p 30

(2) *Journ. Asiat. Soc. Beng.*. viii (1912), p. 357 ; *Bakh. Ms.*, §§ 42, 60.

(3) Compare such expression as *mulam na jnayate*, मूलं न ज्ञायते (folio 13, verso; 15 verso, *prathamam na janami*) प्रथमं न जानामि (24, verso), *padam na jnayate* पदं न ज्ञायते (54, verso), etc. in each case of which the *ajnata*, अज्ञात (unknown) element has been indicated in the statement by *sunya*, शून्य.

(4) Folio 25, verso ; and folio 26, recto,

hypothesis will be further supported by the fact that the similar use of the 'zero' sign to denote the unknown element in the statement (*nyasa*) of problems is found in the arithmetics of Sridhara[1] and Bhaskara[2]. Thus we have[3],

आदि: 20 | उ 0 गच्छ: 7 | गणितम् 245 |

which is a statement of an arithmetical progression whose first term is 20, number of terms is 7, sum is 245 and whose common difference is not known. Both these writers have well-defined notations for the unknown, and do never use the cipher in this way in their treatises on algebra. But as the use of algebraic symbols is not permissible in arithmetic, they make use of the cipher to indicate that certain element in a problem is wanting. Of course, the cipher has wider use in the Bakhshali mathematics than in any of these works.

The lack of an efficient symbolism is bound to give rise to a certain amount of ambiguity in the representation of an algebraic equation, especially when it contains more than one unknown[4]. For instance, in[5]

0	5	यु	मू	0	सा	0	7+	मू	0
1	1			1		1	1		1

which denotes $\sqrt{x+5}=s$, $\sqrt{x-7}=t$, different unknowns will have to be assumed at different vacant places. Again in the statement[6].

आ	5	उ	6	प	0	ध	0
	1		1		1		1
आ	10	उ	3	प	0	ध	0
	1		1		1		1

which refers to two arithmetical progressions whose first terms and common differences are different but known, and whose

(1) *Trisatika*, pp. 19 et seq.

(2) '*Lilavati*', pp. 18 seq. This is not evident from Colebrooke's translation of the works where the cipher has been replaced by the query.

(3) *Trisatika*, p. 29.

(4) Nearly similar difficulty and inconvenience were experienced by the Greek algebraists who had only one symbol for the unknown.

(5) Folio 59, recto. Hoernle and Kaye are not right in thinking that this statement represents :

$$x+5=s^2 \text{ and } x-7=t^2.$$

(6) Folio 5, recto.

sums and number of terms are equal but unknown, $\overset{0}{1}$ stands in the place of two different unknowns[1]. To avoid such ambiguity, in one instance which contains as many as five unknowns, the abbreviations of ordinal numbers such as *pra*, प्र (abbreviated from *prathama*, प्रथम 'first'), *dvi* द्वि (from *dvitiya*, द्वितीय, 'second'), *tr*, तृ (from *trtiya*, तृतीय, 'third'), *ca* च (from caturth, चतुर्थ, 'fourth') and pam, पं (from pancama, पञ्चम, 'fifth') have been used to represent the unknowns; e g.[2].

9 प्र	7 द्वि	10 तृ	8 च	11 पं	युत जातं प्रत्येक
7 द्वि	10 तृ	8 च	11 पं	9 प्र	(क्रमेण)
					16 | 17 | 18 | 19 | [20]

which means,

$$x_1+x_2=16,\ x_2+x_3=17,\ x_3+x_4=18,\ x_4+x_5=19,$$
$$x_5+x_1=20.$$

The want of a proper symbol for the unknown eventually leads to the adoption of the method of 'false position' or 'supposition' for solution of algebraic equations. The solution generally begins with putting 'any desired quantity' (यदृच्छा *yadrccha*) in the vacant place[3].

---

(1) It is not easy to say what is intended to be implied by placing the unity below the cipher. It is supposed by some to be an indication that the unknown quantity will be an integer (*Kaye*, ***Bakh. Ms.***, § 60). Such a supposition is quite untrue. For in the instance cited while *dhana* is an integer (=65), *pada* is a fraction (=13/3).

Strangely this very statement has been quoted by Kaye just after the remark referred to. In certain instances, it is a mixed surd (vide folio 6 and 45, rectos).

(2) Folio 27, verso. In one instance in Bhaskara's *Bijaganita*, initial syllables of the names of particular things have been used as symbols for the unknowns. (Colebrooke, *Hindu Algebra*, p. 195; compare also p. xi. cf. अत्र रूपाणामव्यक्तानां चाद्याक्षरा न्यपलक्षणार्थं; *Bija.*, p 2,

(3) *Yadrccha pinyase sunye* यदृच्छा पिन्यासे शून्ये or *yadrccha vinyase sunye*, यदृच्छा विन्यासे शून्ये that is 'putting any desired quantity in the vacant place' (Folios 22, verso, and 23, recto). On another occasion it is said *Kamikam sunye pinyastam*, कामिकं शून्येपिन्यस्तं or 'the desired quantity is placed in the vacant place (Folio 23, recto and verso). We have also such expressions as sunya-sthane rupam dattava, शून्य स्थाने रूपं दत्त्वा or 'putting one in the vacant place' (Folios 25, verso and 26, recto; compare also folio 22, verso).

Plan of Writing Equations

In the Bakhshali mathematics two sides of an equation are written down one after the other in the same line without any sign of equality being interposed. Thus the equations,

$$\sqrt{x+5}=s, \quad \sqrt{x-7}=t,$$

appear as[1]

| 0 5 यु मू 0 | सा 0 7+ मू 0 |
| 1 1 1 | 1 1 1 |

The equation,

$$x+2x+3\cdot3x+12\cdot4x=300$$

is stated as[2]

| 0 | 2 1 | 3 3 | 12 4 | दृश्य 300
| 1 | 1 1 | 1 1 | 1 1 | 1

Sometimes the unknown quantity is not indicated. Thus the equation

$$\frac{x}{2}+\frac{x}{3}+\frac{x}{4}=65,$$

is represented as[3]

| 1 | 1 | 1 | दृश्य 65 |
| 2 | 3 | 4 | 1 |

This latter plan is followed in the arithmetical treatise of Sridhara and Bhaskara. According to the former[4]

| 1 | 1 | 1 | दृश्य 2
| 2 | 6 | 12 |

means

$$x-\left(\frac{x}{2}+\frac{x}{6}+\frac{x}{12}\right)=2.$$

Bhaskara does not use the lines[5]. It will be noticed that in all the aforementioned works the absolute term is called *drsya*, (दृश्य) meaning 'visible' which is sometimes abbreviated into *dr* (दृ). A distinction is sometimes made in its connotation in the different works. " The problems in connection with which the above equations arise are of the same kind in all the works. But in the Bakhshali work, the term *drsya* (दृश्य) refers to the 'gives' while in the other works it generally refers to the 'remains'[6]. There is, however, one

(1) Folio 59, recto.

(2) Folio 23, verso. See also folios 21, 23, 24, recto, etc.

(3) Folio 70, recto and verso (C). See also folio 69, verso.

(4) *Trisatika*, pp. 13 et seq

(5) *Lilavati*, pp. 11 et seq.

(6) The term *drsya* occurs also in the (GSS), (IV, 4) in the sense of "remainder",

instance in the *Lilavati* in which the connotation of *drsya* is exactly the same as in the Bakhshali work[1]. This term is closely related to *rupa* (रूप), meaning 'appearance', which is the same for the absolute term in the Indian algebra. We find thus the true significance of the Indian name for the absolute term in an algebraic equation. It represents the visible or known portion of the equation while its remaining part is practically unknown or invisible.

The above plan of writing equations differs much from the plan found in ancient Indian algebra in which, (1) two sides are usually written one below the other without any sign of equality and (2) the terms of similar denominations are written below one another, the terms of absent denominations from either sides being indicated by putting zero as its co-efficient (*Datta*).

Certain Complex Series

As already stated, the author of the Bakhshali mathematics is well acquainted with rule for the summation of series in arithmetical progression. Indeed he gives considerable importance to its treatment. There are instances of the geometrical progression in the work[2]. There are further elementary cases of a certain class of complex series, the law of formation of which is quite clear. If a_1, a_2, a_3,......denote the succesive terms of any series, we find series of the type[3],

---

(1) *Lilavati*, p. 11.

(2) | 1 | 3 | 9 | 27 | 81 | दृ $\frac{329}{1}$ |

करणम् । उत्तर···तत्रोत्तर राशीनां योग 87 एष धना दृष्या शोधनीया जाता 242... | पुरुष | 1 | 3 | 9 | 27 | 81 | योग | 121 | अनेन जाता | 2 | एष द्वौ प्रथमस्य धनम् | 2 | 6 | 18 | 54 | 162 | उत्तरराशी संयुतं जातं

| $\frac{2}{1}$ | $\frac{15}{2}$ | $\frac{48}{2}$ | $\frac{147}{2}$ | $\frac{444}{2}$ | एषां···

—Folio 51, verso,

This appears that it represents a double series :

$$\left.\begin{array}{l} t_1 + 3t_1 + 3^2t_1 + 3^3t_1 + 3^4t_1 \\ 0 + \frac{3}{4}t_1, + \frac{3}{4}(t_1+t_2) + \frac{3}{4}(t_1+t_2+t_3) + \frac{3}{4}(t_1+t_2+t_3+t_4) \end{array}\right\} = 329$$

Set $t_1 = 2$, then the first series becomes 242 and the second 87, and the combined series, $2 + \frac{15}{2} + \frac{48}{2} + \frac{147}{2} + \frac{444}{2} = 329$.

(3) The series of these types occur respectively on folio 22, verso ; 23, recto ; 23, recto and verso ; 25, verso and 26, recto : 24, verso, and 25, recto ; 51, recto and verso.

(1) $a_1+2a_1+3a_1+4a_1+\ldots\ldots\ldots+na_1$,

(2) $a_1+2a_1+3a_2+4a_3+\ldots\ldots na_{n-1}$,

(3) $a_1+2a_1+3(a_1+a_2)+4(a_1+a_2+a_3)+\ldots$

(4) $a_1+(2a_1\pm b)+\{3a_1\pm(b+d)\}+\{4a_1\pm(b+2d)\}+\ldots\ldots$

(5) $a_1+(2a_1+b)+\{3a_2+(b+d)\}+$
$\{4a_3+(b+2d)\}+\ldots\ldots\ldots$

(6) $a_1+(2a_1+b)+\{3(a_1+a_2)\pm(b+d)+$
$\{4(a_1+a_2+a_3)\pm(b+2d)+\ldots$

(7) $a_1+(a_1r+da_1)+\{a_1r^2+d(a_1+a_1r)\}+$
$\{a_1r^3+d(a_1+a_1r+a_1r^2)\}+\ldots\ldots$

Evidently the series (4), (5), (6) are obtained respectively from the series (1), (2), (3) with the help of the subsidiary series in arithmetical progression,

$$b+(b+d)+(b+2d)+\ldots$$

Systems of Linear Equations

Examples of the following type occur :

$x_1+x_2=a_1$, $x+x_3=a_2,\ldots\ldots\ldots x_n+x_{n+1}=a_n$ where n is always odd. We have,

$$a_n=(a_2-a_1)+\ldots\ldots+(a_{n-1}-a_{n-2})+2x_1\ ;$$

and if we assume $x_1=p$

then, $a_n'=(a_2-a_1)+\ldots\ldots+(a_{n-1}a_{n-2})+2p$.

Subtracting we get, $x_1=p+(a_n-a'_n)/2$, which is the solution employed in the text.

The following examples occur

(i) $x_1+x_2=13$, $x_2+x_3=14$, $x_3+x_1=15$.

The value of x_1 is assumed to be 5, then by 'subtraction order' $x_2=8$, $x_3=6$ and $x_3+x_1=11$. The correct values are therefore, $x_1=5+(15-11)\div2=7$, $x_2=6$ and $x_3=8$.

(ii) $x_1+x_2=16$, $x_2+x_3=17$, $x_3+x_4=18$, $x_4+x_5=19$,
$x_5+x_1=20$.

Here the value of x_1 is assumed to be 7 and $x'_5+x'_1=16$ therefore the correct values are $x_1=9$, $x_2=7$, $x_3=10$, $x_4=8$, $x_5=11$.[1]

The following are implied,

---

1. Since in these two particular examples the values of a_1 a_2, etc. are in arithmetical progression a, simpler solution would be $x_1=\frac{m}{2}$ where m is the mean of the series a_1, a_2 etc.

(iii) $x_1+x_2=9$, $x_2+x_3=5$, $x_3+x_1=8$.

(iv) $x_1+x_2=70$, $x_2+x_3=52$, $x_3+x_1=66$.

(v) $x_1+x_2=1860$, $x_2+x_3=1634$, $x_3+x_1=1722$.

(vi)[1] $x_2+x_3+x_4+x_5=317$

$x_1+x_3+x_4+x_5=347$

$x_1+x_2+x_4+x_5=357$

$x_1+x_2+x_3+x_4=365$

$x_1+x_2+x_3+x_5=362$

and the following, which, however, is too mutilated to be sure of

(vii) $x_1+x_2=36$, $x_2+x_3=42$, $x_3+x_4=48$, $x_4+x_5=54$, $x_5+x_1=60$.

The directions seem to indicate that we should cancel by six. We then get,

$a'_1=6$, $a'_2=7$, $a'_3=8$, $a'_4=9$, $a'_5=10$ of which the solution is $x'_1=4$, $x'_2=2$, etc., whence $x_1=24$, $x_2=12$, $x_3=30$, etc.

Further Examples

The next set of examples can be represented in the form $\Sigma x-x_r=c-d_1x_1$, $\Sigma x-x_2=c-d_2x_2, \ldots \Sigma x-x_n=c-d_nx_n$.

If we set, $a=1-d$, these become,

1. This can be arranged in the form $y_1+y_2=365$, $y_2+y_3=347$, $y_3+y_4=362$. $y_4+y_5=317$, $y_5+y_1=357$. Solving this we get $x_1+x_2=210$, $x_2+x_3=170$, $x_3+x_4=155$, $x_4+x_5=147$, $x_5+x_1=192$, which solved gives $x_1=120$, $x_2=90$, $x_3=80$, $x_4=75$, $x_5=72$. (See folio 1) Also note that most of these six examples can be expressed in the form $\Sigma x-x_1=a_1$, $\Sigma x-x_2=a_2 \ldots$ $\Sigma x-x_n=a_n$. The examples given in the text are undoubtedly akin to the 'Ephanthema', usually attributed to Thymaridas, which may be expressed by $x_0=\dfrac{\Sigma a-\Sigma x}{n-1}$ where $x_0+x_1=a_1$, $x_0+x_2=a_2 \ldots x_0+x_n=a_n$. In Aryabhata's *Ganita* is a similar rule $\Sigma x=\dfrac{\Sigma d}{n-1}$ where $\Sigma x-x_1=d_1$, $\Sigma x-x_2=d_2, \ldots$ $\Sigma x-x_n=d_n$, which Cantor (*Vorlesungen uber der Geschichte der Mathematik* I, 624) considers to be a modification of the rule given by Thymaridas. See also P. Tannery *Memoires Scientifiques*, Tome II, pp. 192—195.

$\Sigma x - a_1x_1 = \Sigma x - a_2 x_2 = \ldots = \Sigma x - a_n x_n = c$, and $a_1x_1 = a_2x_2 = \ldots a_nx_n = \Sigma x - c = k$.

Therefore $\Sigma x = \left(\frac{1}{a_1} + \frac{1}{a_2} + \cdots + \frac{1}{a_n}\right) k = \frac{p}{q} k.$

A solution is $x_1 = \frac{q}{a_1}, x_2 = \frac{q}{a_2}, \ldots x_n = \frac{q}{a_n}$ and $c - p = q'$.

There are four examples illustrating this process which may be tabulated thus :—

S. No	d_1	d_2	d_3	d_4	d_5	$p-q$	c	x_1	x_2	x_3	x_4	x_5
(i)	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{5}$	$\frac{1}{6}$	437 − 60	377	111	91	81	71	71
(ii)	$-\frac{7}{12}$	$-\frac{3}{4}$	$-\frac{11}{6}$			$\frac{2118}{1463}$	1095	914	836	798		
(iii)	1	7	8				161	41	18	14		
(iv)	2	3	4				17	6	3	1		

Of these only fragments remain. For example, of the first we have;[1]

(*1*) Compare with this the treatment of the *epanthem* by Jamblichus (*Heath, Greek Mathematics*, 1, 93-94

(1) The nearest approach to this that I (Kaye) have came across in an Indian work is given by Mahavira in his *GSS*, VI. 239-240).

Five merchants saw a purse of money. They said one after another, by obtaining $\frac{1}{6}, \frac{1}{7}, \frac{1}{8}, \frac{1}{9}, \frac{1}{10}$ of the contents of the purse I shall become three times as rich as all of you.

Let p be the value of the purse and x_1, x_2 etc., the original capitals:

then $3\ (\Sigma x - x_1) = \frac{p}{6} + x_1, 3\ (\Sigma x - x_2) = \frac{p}{7} + x_2$, etc. whence

$$11 \Sigma x = \left(\frac{1}{6} + \frac{1}{7} + \frac{1}{8} + \frac{1}{9} + \frac{1}{10}\right) p = \frac{6508p}{10080}, \text{ or}$$

$\Sigma x = \frac{6508p}{110880}$. If $p = 110880$, then $x_1 = 261$.

But al-Karkhi (XIth cent.) gave (iii, 6) the following :—

A certain sum is divided among three peopie, one-half being given to the first, one-third to the second and one-sixth to the third. But then one-half the share of the first, one-third the share of the second and one-sixth that of the third is pooled and shared equally by the three.

(*Contd. on next page*)

$$\frac{1}{2} \quad \frac{1}{3} \quad \frac{1}{4} \quad \frac{1}{5} \quad \frac{1}{6} \quad \frac{120}{60}+\frac{90}{60}+\frac{80}{60}+\frac{75}{60}+\frac{72}{60}=\frac{437}{60}$$

$$90+80+75+72=317, \ 317+\frac{120}{2}=377$$

$$120+80+75+72=347, \ 347+\frac{90}{3}=377$$

$$120+90+75+72=357, \ 357+\frac{80}{4}=377$$

$$120+90+80+72=362, \ 362+\frac{75}{5}=377$$

$$120+90+80+75=365, \ 365+\frac{72}{6}=377$$

The second of these examples may be expressed in the form

$$x_1+x_2-(\tfrac{1}{3}+\tfrac{1}{4})\,x_3=x_2+x_3-(\tfrac{1}{4}+\tfrac{1}{2})\,x_1=x_3+x_1-(\tfrac{1}{2}+\tfrac{1}{3})\,x_2$$

From this $\frac{19}{12}x_1=\frac{7}{4}x_2=\frac{11}{6}x_3$ and $\frac{\Sigma x}{\Sigma x-c}=\frac{12}{19}+\frac{4}{7}+\frac{6}{11}$ $=\frac{2558}{1463}$. The solution given is $\Sigma x=2558$ and $c=2558-1463$ $=1091$, whence $x_1=924$, $x_2=836$, $x_3=798$.

Of the third, there is sufficient of the formal question preserved to enable it to be restored.

"One possesses seven horses, another nine mules (?), and a third ten camels. Each gives one of his animals to each of the others and then their possessions become of equal value".

In. A. D. 1225 this problem became famous as it was one of those propounded to Leonardo of Pisa and solved by him.

If x_1, x_2, x_3, be the shares and $3c$ be the amount pooled, then,

$$x_1-\frac{x_1}{2}+c=\frac{\Sigma x}{2}, \quad x_2-\frac{x_2}{3}+c=\frac{\Sigma x}{3},$$

$$x_3-\frac{x_3}{6}+c=\frac{\Sigma x}{6} \text{ and } \Sigma x-x_1=2c, \ \Sigma x-2x_2=3c,$$

$\Sigma x-5x_3=6c$, whence $\Sigma x=\frac{47c}{7}$. Make $c=7$ and

$x_1=33$, $x_2=13$, $x_3=1$.

The fourth example of similar form[1].

Mutilated Examples :

The following occurs on some very mutilated scraps that have been pieced together. It is now impossible to say how it originated or what its context was.

$$x\,(x+y+z)=60$$
$$y\,(x+y+z)=75$$
$$z\,(x+y+z)=90$$

We have $(x+y+z)^2=225$ whence $x+y+z=15$, and $x=4$, $y=5$ and $z=6$.

Solution in Positive Integers :

There is one nearly complete statement and solution of the following pair of equations :—

$$x+y+z=20$$
$$3x+\tfrac{3}{2}y+\tfrac{1}{2}z=20$$

of which the only solution in positive integers is $x=2, y=5$, and $z=13$.

This type of problem was a favourite in Europe and Asia in early mediaeval times. Later it was known as the 'Regula Virginum', 'Regula Potatorum', etc. It was given by Chang-ch' iu-chien (sixth century A. D.)[2], by Alcuin the Englishman (eighth century) and others. About 900 A. D. it was pretty fully treated by Abu Kamil al-Misri who gives some six problems, varying from three to five terms and attempts to find all the positive integral solutions[3].

---

(1) This set of examples is introduced by a rule which simply means : change the fraction A/B into B/ (B – A). This illustrates the fact that the sutras or rules often makes no pretence of indicating in any way the general theory : they are merely intended to be helpful.

(2) Yoshio Mikami.

(3) H. Suter, *Das Buch der Seltenheit*, etc. Bib. Math., (1910—11), pp. 100—120. Suter's *Die Mathematiker und Astronomen der Araber und Ihren Worke*, p. 43.

In the earlier Indian works problems of this type do not occur, but exactly the same example is given by both Mahāvira[1] and Bhaskara II[2].

Quadratic Indeterminate Equations.

There are two types of quadratic indeterminates preserved.

First Type : (i) $x+a=s^2$, $x-b=t^2$.

The solution given may be represented by

$$x=\left[\frac{1}{2}\left(\frac{a+b}{c}-c\right)\right]^2+b$$

which makes both $x+a$ and $x-b$ perfect squares. In the actual example preserved $a=5$ and $b=7$ and the solution is $x=11$, which is the only possible integral (positive) solution obtainable from the formula. No general rule is preserved but the solution itself indicates the rule. It proceeds by steps thus :

$5+7=12$, $12\div 2=6$, $6-2=4$, $4\div 2=2$, $2^2=4$, $4+7=11$.

The value of this type of detailed exposition is here selfevident. It seems to have been almost necessitated by the absence of a suitable algebraic symbolism[3].

Second Type : (ii) $xy-ax-by-c=0$

(1) *GSS.*, vi, 152.
(2) *Vija-Gan.*, 158.
(3) This type of equation occurs in many medeiaeval works from the time of Diophantus onwards (c. g., see Diophantus, ii, 11ff : Brahmagupta, xviii, 84: Al-Karkhi, p. 63, etc.) and has here no very special interest beyond the indication it gives that the Bakhshali text followed the fashion. Dr. Hoernle, however, thought that it indicated a 'peculiar' connexion between the Bakhshali Ms. and Brahmagupta's work : and from this deduced that our text 'may have been one of the sources from whence the later astronomers took their arithmetical information.' (*Indian Antiquary*, xvii, 1888, p. 37).

The solutions which appear to be followed in the text are $x=(ab+c) \div m+b$, $y=a+m$; or $y=(ab+c)\div m+a$, $x=b+m$, where m is any assumed number. The only example that is preserved[1] is $xy-3x-4y+1=0$ of which the solutions given are 15 and 4, and 16 and 5, i. e.,

$$(3\cdot 4-1) \div 1+4=15,\ 3+1=4;\ (3.4+1) \div 1+3=16,\ 4+1=5.$$

Happy Collaboration of the East and the West

In chemistry, astronomy, mathematics, medicine and surgery, there was always in our history a happy collaboration in the advancements, in which all great nations of the East, Middle-East and the West took part. The Bakhshali manuscript in its original form (agreeing to Dr. Hoernle's suggestion) must have had its authorship in the early centuries of our era. From generation to generation, it must have been copied out with alterations and additions. By this time the western mathematics had incorporated almost all that could be traceable to Greek sources, and had assumed new forms and included some new notions also. On the whole the western mediaeval mathematics tended to become less rigorous and but more

1. Unfortunately the text 'folio 27, recto' is so mutilated that the correctness of the interpretation here given cannot be guranteed. But the equation was well known to mediaeval mathematicians and has historical interest. Brahmagupta gave a general solution (xviii, 61) which, however, he appears to have thought unnecessary. Bhaskara also gives a general solution (*Bija Ganita*, 212-214) together with demonstration in both algebraical and geometrical forms; but he also on another occasion (ib. 208-209) gives an arbitrary solution, The general solution is also given by Al-Karkhi. Mahavira gives (vi, 284) the equation in a different form, namely : $\left(a+\frac{1}{x}\right)\left(b+\frac{1}{y}\right)=ab+A$ which reduces to $xy-\frac{a}{A}x-\frac{b}{A}y-\frac{1}{A}=0$ of which the solution is $x=\frac{b}{A}+\frac{1}{m}\left(\frac{ab+A}{A^2}\right)$, $y=\frac{a}{A}+m$; or $y=\frac{a}{A}+\frac{1}{m}\left(\frac{a+b\ A}{A^2}\right)$, $x=\frac{b}{A}+m$. If we make $m=\frac{ab}{A}+1$ then $x=\frac{b+1}{A}$, $y=\frac{a+ab+A}{A}$ which is the solution given by Mahavira.

His examples are : The product of 3 and 5 is 15, and the required product is 18 or 14. What are the quantites to be added or subtracted ? i.e. (i) $(3+1/x)\ (5+1/y)=15+3$, (ii) $(3-1/x_1)\ (5-1/y_1)=15-1$. His answers are (i) 2 and 7, (ii) 6 and 17.

mixed than the mathematics of the classical Greece. Practical calculation (logistic) altogether supported the earlier pure arithmetic; mensuration took the place of pure geometry, and algebra slightly developed. Such changes are, indeed, indicated in the later Alexandrian works, and what we sometimes term degeneration had already set in there. The introduction of a place-value arithmetical notation made calculation easier and more popular, and more intricate arithmetical problems than those exhibited in the Greek Anthology, appeared in the later mediaeval text-books. The general body of popular mathematical knowledge became more diffused. (Identical problems occur in Chinese, Indian, Arabic and European text-books of a comparatively early period). Indeed the mediaeval mathematical works of Asia and Europe had so much in common that at first it seems almost impossible to pick out that which is definitely western or eastern in origin. In this connexion it should be remembered that the early astronomers (Aryabhata II, and Varaha-Mihira, an astronomer of Greek or middle-east origin, settled and naturalized in India,) were among the first to collaborate with Greeks in mathematical and astronomical learning; that later the Arabs, after sampling Indian works, entered into this collaboration; and that it was from the Arabs that Europe received once more the learning it had previously rejected. (Kaye)

We give here a summary chronological table of the period, that may serve to recall the chief mathmatical writers and their works :

Before Christ—Rgvedic period		Earlier than 4000 B. C.
	Taittiriya Samhita	2000 B. C.
	Satapatha Brahmana	1000 B.C. "
	Baudhayana Sulba Sutra	500 B. C.
	Vedanga Jyautisa, Lagadha	500 B. C.
2nd to 3rd Century	Bakhshali Manuscript (original)	300 A. D.
5th Century	Hypatia	d.415 A. D.
	Proclus	410-485 A. D.
	Boethius	b.470
	Aryabhata	b.476

6th Century	Bhaskara I	522
	Eutocius	
	Damascius	529
	Simplicius	
	Dominus	
	Chang Ch'-iu-Chien	550
	Varahamihira	505—587
	Isidore of Seville	570—636
7th Century	Brahmagupta	b.598
	Fall of Alexandria	640/1
	Papyrus of Akhmin	
	Bede	b.735.
8th Century	Muhammad b. Musa	
	Alcuin	b.804
	Sridhara	750
9th Century	Mahavira	850
	Tabit b. Qorra	835—901
10th Century	al-Battani	
	Aryabhat II	c. 950
	Avicenna	
	Pope Sylvester II	d. 1003
11th Century	Albiruni visited India	1017—1030
	al-Karkhi	
	Psellus	
	Omar Khayyam	b. 1046
12th Gentury	Adeland at Cordova	1120
	Bhaskara II	b. 1114
	Leonardo	b. 1175

For the contents of this chapter the author expresses his indebtedness to the critical account of the Bakhshali Mathematics given by Kaye in his publications and to the critical paper of Dr. Bibhutibhushan Datta in the Bulletin of the Calcutta Mathematical Society, 1929.

APPENDIX I

WEIGHTS AND MEASURES

The measures exhibited in the manuscript are of rather special interest. As a whole they are Indian and the terminology is Sanskrit; but there are some Sanskritized western terms such as lipta, dramma dinara, satera employed. Most of the terms are well defined but the, values of some are doubtful. Money measures, however, are, as in most early Indian works, very ill-defined and hardly show any differentiation from measures of weight.

Change ratios : The change ratios are often given with considerable care and elaboration, and are expressed in several different ways. The change ratio appears to be considered as a divisor for it is most frequently marked by the term *chhedam* which indicates the operation of division.

Examples are :

chhe° 80 rakti°—su° i. e. 80 raktika = 1 suvarna

chhe° 24 am°—ha° i. e. 24 angula = 1 hasta

chhe° 2 gha°—mu° i. e. 2 ghatika = 1 muhurta

chhe° 4608000 ya°—yo° i. e. 4608000 yava = 1 yojana

urdha chchhe° 768000 a°—yo°, i.e. 768000 angula = 1 yojana, and in this particular example the operation of multiplication is to be performed.

urdha chchhedam 108000 viliptanam rasi, i.e. 108000 vilipta = 1 rasi and multiplication is indicated.

adha chchhedam 2000 pa°—bha°, i.e. 2000 pala = 1 bhara and division is indicated.

Another form is illustrated in the following examples :

tolenasti dhane 12, i.e. 1 tola = 12 dhana.

dhanenasti am° 4, i.e. 1 dhana = 4 andika

dinaranasti dhane 12, i.e. 1 dinara = 12 dhana

Abbreviations :

The Bakhshali abbreviation for weights and measures are usually as follows :

Adh, आ = Adhaka, आढक

Am, अं = Amsha, अंश

Am, अं = Andika, अण्डिका

Am, अं = Angula, अंगुल

Bha, भा = Bhara, भार

Dha, धा = Dhanaka, धानक

Dha, ध = Dhanusha, धनुष

Di, दि = Dina, दिन

Di, दी = Dinara, दीनार

Dram, द्र = Drama, द्रम्म

Dra, द्र = Drankshana, द्रंक्षण

Dro द्रो = Drona, द्रोण

Mu, मू = Mudrika, मूद्रिका

Mu, मु = Muhurta मुहर्त्त

Pa, पा = Pada, पाद

Pa, प = Pala, पल

Pra, प्र = Prastha, प्रस्थ

Pra, प्र = Prasriti, प्रसृति

Ra, र = Raktika, रक्तिका

Ra, रा = Rasi, राशि

Gav, गव = Gavyuti, गव्यूति

Gha, घ = Ghatika, घटिका

Gum, गुं = Gunja, गुञ्जा

Ha ह = Hasta, हस्त

Ka, क = Kakini, काकिनी

Ka, क = Kala, कला

Kha, ख = Khari, खारि

Kro, क्रो = Krosa, क्रोश

Ku, कु = Kudava, कुडव

Li. लि = Lipta, लिप्ता

Ma, म = Masa, मास

Ma, मा = Mashaka, माशक

Sa, स = Satera, सतेर

Si, सि = Siddhartha, सिद्धार्थ

Su, सु = Suvarna, सुवर्ण

To, तो = Tola, तोला

Va, व = Varsha, वर्ष

Vi, वि = Vilipta, विलिप्ता

Ya, य = Yava, यव

Yo, यो = Yojana, योजन

Time Measures

1 Varsha = 12 Masa = 360 Dina

1 Dina = 30 Muhurta = 60 Ghatika

Arc Measures

1 Rashi = 30 Ansha = 1800 Lipta = 108,000 Vilipta

1 Ansha = 60 Lipta ($\lambda \in \pi \nu \eta$)

1 Lipta = 60 Vilipta

Money Measures

1 Suvarna = $1\frac{1}{3}$ Dinara = $2\frac{2}{3}$ Dramkshana = 16 Dhanaka = 80 Raktika

1 Dinara	= 2 Dramkshana
1 Dramkshana	= 6 Dhanaka
1 Dhanaka	= 5 Raktika

Weight Measures

1 Bhara	= 2000 Pala
1 Pala	= 8 Tola
1 Tola	= 2 Dramkshana
1 Dramkshana	= 6 Dhanaka
1 Dhanaka	= 4 Andika
1 Andika	= $1\frac{1}{4}$ Raktika
1 Raktika	= $3\frac{1}{5}$ Yava
1 Yava	= $2\frac{1}{2}$ Siddhartha
1 Siddhartha	= $2\frac{1}{2}$ Kala
1 Kala	= 4 Pada
1 Pada	= 4 Mudrika

Length Measures

1 Yojana	= 2 Gavyuti
1 Gavyuti	= 8 Krosha
1 Krosha	= 1000 Dhanusha
1 Dhanusha	= 4 Hasta
1 Hasta	= 24 Angula
1 Angula	= 6 Yava

Capacity Measures

1 Khari	= 16 Drona
1 Drona	= 4 Adhaka
1 Adhaka	= 4 Prastha
1 Prastha	= 4 Kudava
1 Kudava	= 2 Prasriti
1 Prasriti	= 2 Pala

Arithmetical Problems

INCORPORATED IN

The Bakhshali Text

A—7

It is possible that 52 recto gives parts of the solutions of the example on 51 recto which would make that page the reverse, (of course, doubtful). What is left of the solution means :

$$x\left(\frac{1}{10}+\frac{1}{8}+\frac{1}{4}\right)=57$$

or $$\frac{19}{40}x=57$$

$$\therefore x=120.$$

A proof by the rule of three

$$1 : 120 :: \frac{1}{10} : 12$$

$$1 : 120 :: \frac{1}{8} : 15$$

$$1 : 120 :: \frac{1}{4} : 30$$

ii. The example, which is continued on 52 verso, may be expressed by $x\ (1-\frac{1}{2})\ (1-\frac{1}{4})\ (1-\frac{1}{5})=x-280$, whence $x=400$. A proof follows : $400\times\frac{1}{2}=200$ and $400-200=200$; $200-50=150$; $150-30=120$; and $400-120=280$

Again, $\frac{1}{2}+\frac{1}{4}(1-\frac{1}{2})+(1-\frac{1}{4})(1-\frac{1}{2})=\frac{1}{2}+\frac{1}{8}+\frac{3}{8}=1.$

A—9

29 a, b c recto. The problem and its solution here partly preserved may be represented as

$$x_1+x_2=13;\ x_2+x_3=14;\ x_3+x_1=15.$$

If $x_1=5$, then $x'_2=8$, $x'_3=6$, and hence $x'_3+x_1=5+6=11$. From this, the correct value would be

$$x_1=5+(15-11)-2-'$$

whence $x_2=6$ and $x_3=8$.

The phrase "*shodhayet kramat*" recurs in the next example and is a quotation from a lost *sutra*.

29 d, b, c, verso. The example here given (continued on folio 27 verso) is formulated with exactly the same phraseology as the previous one. It may be represented by :

$x_1+x_2=16$; $x_2+x_3=17$; $x_3+x_4=18$; $x_4+x_5=19$ and $x_5+x_1=20$.
If $x_1=10$, then $x_2=6$; $x_3=11$; $x_4=7$; $x_5=12$ and $x'_5+x'_1=22$

Therefore, the correct value of $x_1=10-(22-20)\div 2=9$, and then $x_2=7$, $x_3=10$, $x_4=8$ and $x_5=11$.

The phrases "*iccha*' and "*shodhayet kramat*" are quotations from a lost *sutra*.

A—10

27 verso gives the answer of the problem given on folio 29 verso, namely $x_1=9$, $x_2=7$, $x_3=10$, $x_4=8$, $x_5=11$ and the sums of the pairs of 16, 17, 18, 19, 20.

27 *recto*. Solution of a lost problem which may have been $xy-3x-4y\pm 1\equiv 0$, of which solutions are :

$$x=\frac{3.4-1}{1}+4=15;\ y=3+1=4;\ x=4+1=5;\ y=\frac{3.4+1}{1}+3=16.$$

The quotations are from a *sutra* very much like the one that follows. The phrase "*prathak rupam vinikshipya*" 'having added unity in each case', appears to be a quotation from a lost *sutra*.

A—11

The example is solved on 1 verso and 2 recto and appears to have been somewhat as follows :

The combined capitals of five merchants less one-half of that of the fifth is equal to the cost of a jewel. Find that cost of the jewel and the capital of each merchant.

A 11. *1 verso*. This appears to give part of the solution and proofs of the question on 1 verso.

Then follows 'proof' which may be expressed by

$$90+80+75+72=317 \text{ and } 317+\tfrac{120}{2}=377$$

$$120+80+75+72=347 \text{ and } 347+\tfrac{90}{3}=377.$$

$$120+90+75+72=357 \text{ and } 357+\tfrac{80}{4}=377.$$

$$120+90+80+72=362 \text{ and } 362+\tfrac{75}{5}=377.$$

$$120+90+80+75=365 \text{ and } 365+\tfrac{72}{6}=377.$$

Compair with *sutra* 11 on 1 recto.

A 12

A 12.2 recto This appears to be another 'verification' of the example on 1 recto et verso; and means :

$$\tfrac{120}{2}+90+80+75+72=377.$$

$$120+\tfrac{90}{3}+80+75+72=377,$$

$$120+90+80+\tfrac{75}{5}+72=377.$$

$$120+90+80+75+\tfrac{72}{6}=377.$$

and 'this is the measure of the price of the jewel'.

A 13

A 13. Example : One possesses 7 horses (a^o = *asva*), another 9 horses (ha^o = *haya*) and a third 10 camels (u^o = *ushtra*). Each gives one of his animals to both the others and then their possessions are of equal value. It is required to find the capital of each merchant or the price of each animal. If thou art able, solve me this riddle.

B 1

B 1.8 verso. i. The *sutra* is partially reconstructed from the quotations in the solutions below.

ii. The example is : There are ten horses of which five are yoked at a time to the chariot. How many changes should there be in a journey of one hundred yojanas and how much will each horse do ?

The solution is $\tfrac{100}{10}=10$ stages and $10 \times 5=50$. Mahavira gives a similar example (vi, 158).

"ravi-ratha-turagas sapta hi chatvaro'sva vahant dhuryuktah yojana-saptati-gatyah ko vyudhah ko chaturyogah". It is well known that the horses of the Sun's chariot are seven. Four horses are yoked at a time. They have to perform a journey of 70 yojanas. How many times are they unyoked and how many times yoked ?

B 2

"For a certain feast one Brahman is invited on the first day, and on every succeeding day one more Brahman is invited. For another feast, 10 Brahmans are invited on every day. In how many days will their numbers be equal; and how many Brahmans were invited ? The use of the term *labdham* is here rather curious. The phrases *labdham dvigunitam kritva, tathadvyunam, uttaram vibhajitam* and *rupadhikam* are probably quotations from a *sutra.*

B 2. 9 verso. The example probably meant : A and B start for a place 70 yojanas distant. A travelled at the rate of 1 yojana a day and B at the rate of 6. At what point on his return journey did B meet A ?

Since, $\frac{x}{1} = \frac{2 \cdot 70 - x}{6}$, where x is the distance traversed by A, we have $x = \frac{2 \cdot 70 - x}{6+1} = 20$ as given in the text, and since A travels at the rate of one yojana a day, this is also the time.

Proof by the 'rule of three'.

1 day : 6 yo° :: 20 days : 120 yo°, and 70−20=50 and 70+50=120. Also 1 day : 1 yo° :: 20 days : 20 yo°.

B 4

B 4. i. One goes at the rate of 5 yojanas for 7 days and then a second starts at the rate of 9 yojanas a day. When will they have traversed eqnal distances ?

The phrase *gatisyaiva vishesam ca* is a quotation from *sutra* 15 (folio 3 recto) and *purva gata* is a reference to the same rule.

The solution is $t = \frac{7 \cdot 5}{9-5} = \frac{35}{4}$ days. 'Proof by the rule of three'

$1 : : 5 : : \frac{55}{5} : \frac{175}{4}$ and $1 : 9 :: \frac{35}{4} : \frac{315}{4} - \frac{175}{4} = 35$

One travels at the rate of 18 yojanas in one day for a period of 8 days. A second goes at the rate of 25 yojanas in one day. Determine in what time......

D 5

D 4 With some uncertainty :

A earns $3\frac{1}{2}$ drammas in 2 days, B earns $2\frac{1}{2}$ in 3 days. A gives B7 drammas and this makes their possessions equal. How long had they been earning ?

Since, $\frac{3\frac{1}{2}}{2}\, t-7=\frac{2\frac{1}{2}}{3}\, t+7$, we have $t=\frac{14}{\frac{7}{4}-\frac{5}{6}}=\frac{168}{11}=15\frac{3}{11}$ days.

And 3 days : $2\frac{1}{2}$ drammas: : $\frac{168}{11}$ days : $\frac{294}{11}$ drammas and

$\frac{294}{11}-\frac{77}{11}=\frac{140}{11}+\frac{77}{11}$ days : $\frac{140}{11}$ drammas

ii. Another example of the same kind.

D 6

D 6. 67 recto. i. The example seems to relate to a game at which a certain quantity was staked and eventually all lost. The statement means,

$$1+\frac{1}{2}\left(2+\frac{1}{2}\left(3+\frac{2}{2}\left(4+\frac{3}{2}\left(5+\frac{4}{2}.\ \frac{1}{4}\right)\right)\right)=\frac{23}{18}.$$

E 1

E 1. The problem may have been something like this : The rates of purchage are one, two, three, four and six articles for one drama.

The cost of one of each would be $1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{6}=\frac{27}{12}$, therefore the cost of 12 of each is 27 drammas, and the numbers of articles are 12, 6, 4, 3 and 2.

E 2

53 recto. The following conjectural restoration of the problem is offered :

One goes $1\frac{1}{3}$ yojana in a day and another 6 in 3 days. If the first had a start of 9 yojanas, when would the second overtake him ?

53 verso. The following is merely a guess at the problem : One goes 18 yojanas in 96 days and another 27 yojanas in 108 days. The first starts from A and the second from B and the distance AB is 9 yojanas. When will they meet if they go only for $\frac{7}{12}$ (or 35 ghatikas) of each day ? (60 ghatikas = 24 hours).

E 3

58 verso. There is basis for the following restoration—A man earns 3 in one day, a young woman $1\frac{1}{2}$ in one day and..... $\frac{1}{2}$ in one day. If 20 earn 20 mandas in one, let x, y, z be the numbers of each class. Then, $x+y+z=20$ individuals :

$$3x+\frac{3}{2}y+\frac{z}{2}=20 \text{ mandas}$$

of which the only solution in positive integers is that given in the text, namely $x=2$, $y=5$ and $z=13$. This problem is known as the *Hundred Hens* problem in China, and as the *Requla Virginum*, etc. in Europe.

F 1

• *F 1.22 verso I.* This appears to be the beginning of a new section. This *sutra* is lost.

The problem was something like this : A certain amount given to the first, twice that to the second, thrice it to the third, and four times to the fourth. State the amount given to the first and the shares of the others, if the total amount given was 200.

The shares are represented by 0, 1, 2, 3, 4 'Having added one to the nought' the sum is $1+2+3+4=10$. Then the proper share of the first is $\frac{200}{10}=20$. Having added in this value the series becomes $20+40+60+80=200$. The proof by the *rupona method* gives $\left[(4-1)\frac{20}{2}+20\right]4=200$.

For the meathod of solution, the *regula falsi*, see Part I. Sec. 71 and 72, and for the *rupona method* see Sec.73. The whole section is dealt with in Sec. 87, and the use of the symbol for nought in Sec. 60. (*Kaye*)

ii. The *sutra* beings. "Put what number you please in the empty place (or for the nought)." This is quoted on folio 23 recto and so is *tada vargam tu karayet*, etc.

F 2

F 22, 23 recto. The example may be represented by $x+2T_1+3T_2+4T_3=132$, where T_1, T_2, etc. represent the values of the first, second etc. terms. Make $x=1$, then thc terms are $1+2+6+24=33$ and the proper value of x is $\frac{132}{33}=4$ and the series becomes $4+8+24+96=132$.

All the technical terms here employed are of interest : *ichchha* an assumed number'; *varga* a series; *prakshepa*' something thrown in' or an *interpolation*'; *vartya* 'cancelled'; *drsya* 'the given number' : etc.

ii. The term *kamika* (कामिक) is practically synonymous with *ichchha* or *yadrichchha*' that you please. For an assumed number, Bhaskara uses *ishta* much in the same way. A good deal of the *sutra* is quoted on folio 2, 23 *verso*.

23 verso. i. The example may be represented by $x=2T_1+3(T_1+T_2)+4(T_1+T_2+T_3)=300$. Put $x=1$, then the series becomes $1+2+9+48=60$ and the proper value of x is $\frac{300}{60}=5$ and we have $T_1=5$, $T_2=10$, $T_3=45$, $T_4=240$ and total $T=300$.

ii. The example is solved on folio 24 *recto*.

F 3

F 3. 24 recto. The example may be represented by $x\ (1+1\frac{1}{2})+2T_1+2\frac{1}{2}x+3T_2+3\frac{1}{2}x+4T_3+4\frac{1}{2}x=144\frac{1}{2}$: Set $x=1$ and the series becomes :

$\frac{5}{2}+\frac{15}{2}+\frac{52}{2}+\frac{217}{2}=\frac{289}{2}=144\frac{1}{2}$ which is the same as the given sum and therefore $x=1$ is correct.

24 verso. i. The example may be represented by

$$x\ (1+1\tfrac{1}{2})+2T_1+\tfrac{5}{2}x+3\ (T_1+T_2)+\tfrac{7}{2}x+4\ (T_1+T_2+T_3)+\tfrac{9}{2}x=222.$$

Set $x=1$ and the series becomes $\frac{5}{2}+\frac{15}{2}+\frac{67}{2}+\frac{357}{2}=222$ The same quotation *shunya sthane.. rupam datva* occurs on folio 25 verso. See also at the bottom of folio 26 recto.

F 4.

F 4. *25 recto.* The example may be represented by $x\ (1+\frac{1}{2})+2T_1-\frac{2}{5}x+3\ (T_1+T_2)-\frac{7}{2}x+4\ (T_1+T_2+T_3)-\frac{9}{2}x=78$. Set $x=1$ and the series became $\frac{5}{2}+\frac{5}{2}+\frac{23}{2}+\frac{123}{2}=\frac{156}{2}$ and $x=1$.

25 verso. An example, of which only the solution remains.

ii. The example of which the solution is given on folio 26 recto.

F 5.

F 5. 29 recto. i. This is the solution of the example given at the bottom of folio 24 verso. Let $x=1$, then the series becomes $\frac{5}{2}+\frac{5}{2}+\frac{8}{2}+\frac{11}{2}=\frac{29}{2}$ and the correct value of x is $\frac{29}{2}\div\frac{29}{2}=1$.

G 1.

G 1. 10 verso. i. Gives further proofs of the example on the obverse, namely :

$$x_1\ (1-\tfrac{1}{3})\ (1-\tfrac{1}{3})\ (1-\tfrac{1}{3})=32,\ \text{hence}\ x_1=108;$$

then two proportions in words and figures $\frac{27}{8}$: 1 :: 108 : 32 and a : $3\frac{3}{8}$ :: 32:108.

ii. Example—Of iron once refined three-tenths is lost. What is the remainder of twice seventy, tell me, O' Pandit ? The loss on unity is $\frac{3}{10}$ and the remaineder is $\frac{7}{10}$. The original quantity is 140 and $\frac{7}{10}$ of 140=98. The loss is therefore, 42 and 98+42=140. Proof. $\frac{7}{10}$: 1 :: 98 : 140

Continued on fol. ii. recto.

iii. The example may be rendered :

The third part of the burnt bronze in three instalments (is lost). The amount given was one-hundred and eight. State the remainder, O' Pandit.

The solution according to the rule gives

$108\,(1-\frac{1}{3})\,(1-\frac{1}{3})\,(1-\frac{1}{3})=32$. But proceeding by steps $\frac{108}{3}=36$ and the remainder is 72; $\frac{72}{3}=24$ and the remainder is 48; $\frac{48}{3}=16$ and the remainder is 32.

The proof may be represented by

$$x_1=\frac{32}{(1-1/3)\,(1-1/3)\,(1-1/3)}$$

G. 2

ii. Example—In purchasing one and a half palas the loss is one-third. State what would be the loss on eighteen. Since $\frac{1}{3}/\frac{3}{2}=\frac{2}{9}$, the loss on unity, the remainder is $\frac{7}{9}$. Now $\frac{7}{9}$ of $18=14$ and the loss is 4.

Proof by the rule of three : $\frac{3}{2}:\frac{1}{3}::18:4$ and $\frac{1}{3}:1\frac{1}{2}::4:18$

Example—In refining bronze there is a loss of one-fourth.

What would be the loss on 500 suvarnas four times refined?

The solution is lost. It amounted to

$$500\,(1-\tfrac{1}{4})\,(1-\tfrac{1}{4})\,(1-\tfrac{1}{4})\,(1-\tfrac{1}{4}) = 158\frac{13}{64}\text{su}^{\circ}$$

$$=158\ \text{su}^{\circ}+1\tfrac{1}{64}\ \text{to}^{\circ}$$

(Since 5 tolas = 1 Suvarna)

(*11 verso.*) This appears to have contained five proofs of the example on the obverse, for the present third proof is designated 'the fourth'. The proofs are—

i. Missing.

ii. $x^1\,(1-\frac{1}{4})\,(1-\frac{1}{4})\,(1-\frac{1}{4})=158\ \text{su}^{\circ}+1\frac{1}{64}\ \text{to}^{\circ}$,
therefore, $x^1=500$.

iii. $500\,(1-\frac{1}{4})\,(1-\frac{1}{4})\,(1-\frac{1}{4})=x^1$ and $x=158\ \text{su}^{\circ}+1\frac{1}{64}\ \text{to}^{\circ}$

iv. $x^1=(158\ \text{su}^{\circ}+1\frac{1}{64}\ \text{to}^{\circ})/(1-\frac{1}{4})\,(1-\frac{1}{4})\,(1-\frac{1}{4})$ and $x^1=500$.

v. The first basis is $\frac{500}{4}$ su° = 125 su° and remainder is 375 su°. The second loss is $\frac{375}{4}$ su° = 93su° + 3 to° + 9 masha (since 12 ma° = 1 to°) and remainder is $281\frac{1}{4}$ su°. The third loss is $281\frac{1}{4}/4 =$ $70\frac{5}{16}$ and the remainder is $210\frac{15}{16}$. The fourth loss is $210\frac{15}{16}/4 =$ $52\frac{37}{64}$ and the remainder is $158\frac{13}{64}$.

G. 3.

The example may be conjecturally restored : A traveller goes on a journey of 4 gavyutis and takes with him 4 prasthas of wine. After each gavyuti he drinks 1 prastha and then fills up his bottle with water. How much wine and how much water will there be at the end of his journey ?

G. 3. (12 *verso*). The solution of the example on the obverse is now done by steps. The original amount of 4 prasthas is expressed in kudavas, namely 16.

	Kudavas of wine drunk	Kudavas of wine left	Kudavas of water
Originally	0	16	0
After first gavyuti	4	12	4
After second gavyuti	$\frac{12}{4}=3$	9	7
After third gavyuti	$\frac{9}{4}=2\frac{1}{4}$	$6\frac{3}{4}$	$9\frac{1}{4}$
After fourth gavyuti	$\frac{27}{4}\times\frac{1}{4}=\frac{27}{16}$	$5\frac{1}{4}$	$10\frac{3}{4}$

G 3. 12 verso and 13 recto. Kumkum was purchased for 8, and at each toll office the toll was paid one-fourth. There were four toll-offices, calculate the total toll paid, and the Kumkum left, O' Pandit.

Solution : i. $8\times\frac{3}{4}=6$, and 2 is paid as the first toll.

ii. $6\,(6-\frac{1}{4})=4\frac{1}{2}$, and the second toll is $1\frac{1}{2}$.

iii. $4\frac{1}{2}\,(1-\frac{1}{4})=\frac{27}{8}=3\frac{3}{8}$, and the third toll is $1\frac{1}{8}$.

iv. $3\frac{3}{8}\,(1-\frac{1}{4})=\frac{81}{32}=2\frac{17}{32}$, and the fourth toll is $\frac{27}{32}$.

The total toll paid is :

$$=2+1\tfrac{1}{2}+1\tfrac{1}{8}+\tfrac{1}{2}=\tfrac{1}{32}\,(64+48+36+27)=\tfrac{175}{32}=5\tfrac{15}{32}.$$

$$8-5\frac{15}{32}=\frac{81}{32}=2\frac{17}{32}.$$

G 4.

G 4. (*13 recto*). i. Here are four 'proofs' of the example given on folio 12 verso.

(a) $8(1-\frac{1}{4})\,(1-\frac{1}{4})\,(1-\frac{1}{4})\,(1-\frac{1}{4})=\frac{81}{32}.$

(b) $8.\,\frac{3}{4}.\,\frac{3}{4}.\,\frac{3}{4}.\,\frac{3}{4}=\frac{81}{32}.$

(c) $8(1-\frac{3}{4})\,(1-\frac{3}{4})\,(1-\frac{3}{4})\,(1-\frac{3}{4})=\frac{81}{32}.$

(d) $x^1=\dfrac{81/32}{(1-\frac{1}{4})\,(1-\frac{1}{4})\,(1-\frac{1}{4})\,(1-\frac{1}{4})}$, whence $x^1=8$

ii. There is a quantity of molasses weighing eigut bharakas. What will be left after giving away one-third, one-sixth and one-fifth.

$$8(1-\tfrac{1}{3})\,(1-\tfrac{1}{6})\,(1-\tfrac{1}{5})=\tfrac{32}{9}=3\tfrac{5}{9}.$$

iii. Example—By a gain of five-fourth, ten dronas are obtained Let it be said. O best calculators, what will be the gain by three transactions.

Here the term *labha* seems to have meaning "capital profit", what is termed the 'mixed quantity' *misraka* on folio 62.

(b) $x^1\,(1-\frac{1}{4})\,(1-\frac{1}{4})\,(1-\frac{1}{4})=19\frac{17}{32}$ dro° = 19 dro° + 2a° + O pra° + 2 ku° whence $x^1=10$.

The conversion table of capacity measures is given in folio 13 as follows (marked with asteriks) :

4 adhakas (a°) = 1 drona (dro°)

4 prasthas (pra°) = 1 adhaka

4 kudava (ku°) = 1 prastha.

ii. Example—The capital of a certain banker is sixty, One half of it goes in loss and then he gains by one-third; next he loses one-fourth of it and finally gains one-fifth; so that he has two gains. What is his gain and what is his loss and what the remainder and let that be stated.

Solution : $60\,(1-\frac{1}{2})\,(1-\frac{1}{4})\,(1+\frac{1}{3})\,(1+\frac{1}{5})=36.$

Proofs : (a) $x^1=\dfrac{36}{(1-\frac{1}{2})\,(1+\frac{1}{3})\,(1-\frac{1}{4})\,\,(1+\frac{1}{5})}$.

whence $x^1=60.$

(b) $60\,(1-\frac{1}{2})\,(1+\frac{1}{3})\,(1-\frac{1}{4})\,(1+\frac{1}{5})=36.$

(c) $x^1\,(1-\frac{1}{2})\,(1+\frac{1}{3})\,(1-\frac{1}{4})\,(1+\frac{1}{5})=36.$

whence $x^1=60.$

G—5

G 5. ii. Example—A known amount of molasses equal to four is increased by one-third, one-fourth, one-fifth, one-sixth and then forty is lost.

No solution is preserved.

G 5. (*14 verso*). i. Example—An unknown quantity of lapis-lazuli loses one-third, one-fourth, and one-fifth, and the remainder after the three-fold operation on the original quantity is twenty seven. State what the total was, O wise one, and also tell me the loss.

Solution : $\frac{2}{3}.\,\frac{3}{4}.\,\frac{4}{5}.=\frac{2}{5};\ 1-\frac{2}{5}=\frac{3}{5};\ 27\div\frac{3}{5}=45$ and $45-27=18$ and this is the loss.

The meaning of *ambha-loha,* lapis-lazuli, was suggested by Dr. Hoernle.

ii. Example—Of the loss of iron the third is one-fifth of a masha. The original quantity is not known and neither is the remainder given but only the original remainder which quantity stands as twenty. Tell me what is the unknown original quantity and what is the remainder.

The interpretation, however, is by no means certain. The solution is lost.

G—7

ii. Example—Of a ghataka of honey two-third is given, to the second two-fifth; to the third two-sevenths; to the fourth two-ninths, till only three palas are left). O' Pandit, state how much altogether was taken away by the tax collector.

H—1

H 1. ii. *Rule* : Having multiplied the parts of gold with the *ksaya* let this sum be divided by the sum of the parts of gold. The result is the average *ksaya.* This means :

$$f = \frac{f_1g_1+f_2g_2+\cdots+f_ng_n}{g_1+g_2+\cdots+g_n}$$

where f denotes *ksaya* and g gold.

iii. Example—$f_1=1$, $f_2=2$, $f_3=3$, $f_4=4$, and $g_1=1$, $g_2=2$, $g_3=3$, $g_4=4$, therefore,

$$f = \frac{1.1+2.2+3.3+4\,4}{1+2+3+4} = \frac{30}{10} = 3.$$

H 2

H 2. ii. Example—Gold one, two, three, four; abandoned, the following mashakas one-half, one-third, one-fourth, and one-fifth.

$$F = \frac{\frac{1}{2}.1+\frac{1}{3}.2+\frac{1}{4}.3+\frac{1}{5}.4}{1+2+3+4} = \frac{163}{60} \div 10 = \frac{163}{600}.$$

Proof by the rule of three $g : fg :: g_r : g_rF$.

H 3.

H 3. ii. Example—Mashakas of one and two, gold of two and five, mashkas of three and gold unknown. All that is known is the sum of mashakas, six; and the average mashakas four. State the mashaka of the unknown gold.

Statement : $f_1=1, f_2=2, f_3=3; g_1=2, g_2=5, g_3=x; F=4$

Solution : $\frac{4.6-(2.1+5.2)}{3}=\frac{24-2-10}{3}=\frac{24-12}{3}=\frac{12}{3}=4$

J 1.

J 1. Example—If a man requires six drammas for his livelihood for 30 days, for how many days will 70 men (guards of a fort) live on six drammas ? The details are, however, uncertain.

K 1.

K 1. i. Example—What number with five added is a square and that same number with seven substracted also being a square ? What is that number ? is the question.

Statement : $x+5=s^2,\ x-7=t^2$

Solution : $x=\left[\frac{1}{2}\left(\frac{(5+7)}{2}-2\right)\right]^2+7=11.$; by steps thus :

Having combined the added and subtracted number $5+7=12$: that halved $=6$; two substracted 4; halved 2; squared 4; then that subtractive number (7) is to be added and by the addition of this $4+7=11$; and this is the required quantity.

Proof : $11+5=4^2,\ 11-7=2^2$

For such problems, if $x+a=s^2$ and $x-b=t^2$, then the solution is,

$$x=\left[\frac{1}{2}\left(\frac{a+b}{c}-c\right)\right]^2+b.$$

L 1.

L 1. iii. Example—In two days one earns five; in three days he consumes nine. His store is thirty. In what time will his earnings be consumed ?

Solution : $t=\frac{30}{\frac{9}{3}-\frac{5}{2}}=60$ and the amount earned in this time is $\frac{5}{2}$ of $60=150$ dinaras.

iii. Example—In three days one Pandit earns a wage of five and a second wise man earns six (*rasa*) in five days. The second is given by the first seven from his store and by this giving their possessions become equal. Let it be stated in what time.

Solution : $t=\dfrac{2\times 7}{\frac{5}{3}-\frac{6}{5}}=30.$

L 2.

Profit and loss.

The author gives a number of solved examples on problems concerning profit and loss. The following rules are given with illustrative examples :

[Let C be the capital, p the profit, $M=C+p$, $N=Cc$, the number of articles, C the cost rate, and s the sale rate, $C=a/b$, where a is the number purchased for b drammas (but money measures are not generally stated), and $s=C/d$, where C is sold for d money.]

$$\text{(i) } C=\frac{p}{\frac{c}{s}-1}\ ; \qquad \text{(ii) } p=\frac{Cc}{s}-C$$

$$\text{(iii) } C=\frac{M}{\frac{c}{s}}=\frac{C+p}{\frac{c}{s}}\ ; \quad \text{(iv) } C=\frac{-p}{1-\frac{c}{s}}$$

L 2. ii. Example—Two Rajputs are the servants of a king. The wages of one are two and one-sixth a day, of the second one and one-half. The first gives to the second ten dinaras. Calculate and tell me quickly in what time there will be equality. (*Indian Antiquary*), 1881, p. 44).

Statement : $\frac{13}{6}$, $\frac{3}{2}$, given 10.

Solution : The difference of the daily earnings.

$\frac{13}{6}-\frac{3}{2}=\frac{2}{3}$ dinaras.

Therefore, a difference of 20 dinaras would be in 30 days.

L 2. 6 verso. i. Proof of example on the obverse—

$1 : \frac{13}{6} :: 30 : 65.$

$1 : \frac{3}{2} :: 30 : 45$ and $65-10=45+10.$

ii. The rule means, $C=\dfrac{p}{\frac{c}{s}-1}$, where C is the capital, p the profit, C is cost rate and s is sale rate.

iii. Example—One buys 7 for 2 and sells 6 for three and 18 is his profit. What was his capital ?

Solution : $C=\dfrac{18}{\dfrac{7}{2}+\dfrac{6}{3}-1}=24.$ The proof is given on folio 62 recto.

L 3.

iii Example—Eight articles are obtained for three and six are sold for four. The sum of the capital and profit is one-hundred and sixty. State, O' best of calculators, what was the capital and what is the profit.

The solution is lost except for the first quotation, but part of a proof is given on folio 63, recto. The solution was,

$$C=\frac{160}{\frac{16}{9}}=90.\text{ Since } C=\frac{p}{\frac{8}{3}\,/\,\frac{6}{4}-1}=\frac{p}{\frac{16}{9}-1}=\frac{p}{\frac{7}{9}}$$

$p=\frac{7}{9}C$ and $C+p=160$, and the number of articles bought was $\frac{8}{3}$ of $90=240$.

L 4.

iii. Example—With five four-squared are obtained by some man. For one-six are sold and fifty-six is the loss. Calculating purchase and sale let his capiteal be stated.

The solution is $C=\dfrac{56}{1-\dfrac{16}{30}}=120$ and the number of articles is $\frac{16}{5}$ of $120=384$.

L 4. recto. i. Proof of example given on folio 62 verso. 8:3::240:90 and 6:4::240:160 and 160=90+70.

ii. The rule means $C=\dfrac{l}{1-\dfrac{c}{s}}$, where l is the loss sustained, i e. having investigated the selling rate multiply with the purchase rate and having subtracted from unity divide and the capital is obtained.

L 4. 63 verso. Proof of example on the obverse :

s:1::384:120 then with the selling rate 6:1:384:64 and 120 – 64 = 56.

ii. *Rule*—That whieh is the tax on cloth is taken in cloth : the tax on a piece of cloth is one-twentieth part.

Some one sells three-hundred. On the pieces being brought to market, two pieces are taken by way of tax : ten is (?) the selling price. What is the value ?

M 1.

M I. 20 verso. i. A mere fragment : 12,000 mudrikas...

ii. Example—A snake eighteen hastas long enters its hole at the rate of one-half plus one-ninth; of that, minus one-twenty first part of an angula a day. In what time will it have completely entered its hole ?

Solution : Since 24 angulas = 1 hasta, we have $\frac{1}{2}+\frac{1}{9}$ of $\frac{1}{2}-\frac{1}{21}$: $\frac{1}{380}$ :: 18 × 24 : x, whence $x=\frac{189}{80}$ = 2 years 4 months $10\frac{1}{2}$ days.

M 2.

M 3. 38 recto. i. Example—The earning of dinaras is difficult but consuming them is easy. One gives one-half increased by ratio on one-half (six times) for food for the poor. What is the amount consumed in 108 days ?

Solution : 1 : $\frac{1}{2}.\frac{1}{2}.\frac{1}{2}.\frac{1}{2}.\frac{1}{2}.\frac{1}{2}$ :: 108:1 di^{o}+8dha^{o}+1am^{o} i e, 1:$(\frac{1}{2})^{6}$ :: 108 : 1 di^{o}+8 dha+1 am^{o} and 4 amsas^{o}=1 dhanaka^{o} and 12 dhanaka^{o}=1 dinara^{o}.

ii. Example—(This is not understood, but appears to refer to the number of hairs on the skin of an animal).

M 3

M 3. i. A mutilated example about Ravana and Sita. When Sita had been carried up 30 yojanas into the air she dropped something to earth, which turned over 8 times in $1\frac{1}{10}$ dhanusha How many revolutions did it make before reaching the earth ?

(1 yojana = 8000 dhanusha)

Solution : $(1\frac{1}{8}-\frac{1}{10})$ dha° : 8 revolutions :

30 × 8,000 dhanusha :: 210, 181, 181 $\frac{9}{11}$ revolutions.

ii. Example - A snake which is 100 yojanas, 6 krosas, 3 hastas and 5 hastas and 5 angulas long sheds its skin at the rate of 1 angula in 2 days. In what time will it be free. ?

The solution is given (?) on the reverse.

30 verso. i. $\frac{1}{2}$ 1 an° : 2 d° :: 100 yo°:+6kr°+3 ha°+5an° .

429,867 years 1 month and 4 years.

$$\text{or } 1 : 2 :: 77,376,077 \text{ an}^{\circ} : \frac{154752154}{360} \text{years.}$$

ii. An example about some garment falling to the earth. The elements are uncertain. Compare with the problem on the obverse (1).

M 4.

M 4. 36 recto. i. "This land measurement is completed" may refer to the fragmentary example at the bottom of folio 32 verso but it is doubtful.

ii. The example appears to refer to heaps of salt. If one heap or quantity weighs 1,075 palas how much will 56 heaps weigh ?

1 : 1075 :: 56 : 30 bha°+200 pn°

$$\text{or } \frac{1075 \times 56}{200} = \frac{60200}{2000} \text{ bhara} = 30 \text{ bha}^{\circ} + 200 \text{ pa}^{\circ}.$$

iii. One-tenth of a cowry is givea in eighty eight...of this one twentieth and one-hundredth...

36 verso. i. The statement means 3 yo° : 1 day :: 5 yo° :: 21, 333 years 4 months,

or 3yo° : $\frac{1}{360}$ years :: 5 × 4,608,000 yo° : 21.333 years 4 months.

$$\frac{5 \times 4,608,000}{3 \times 360} = 21,333\frac{1}{3}.$$

ii. A boat goes one-half of a third of a yojana plus one-third less one quarter, $\frac{1}{2}$ of $\frac{1}{3}+\frac{1}{3}-\frac{1}{4}$, yojana in one-half of one-third of a day, but then it is driven back by the wind one-half of one-fifth of a yojana, $\frac{1}{2}$ of $\frac{1}{5}$ yo°, in one eight of three days. In what time will it travel one hundred and eight yojana.

$$\frac{108}{(\frac{1}{3} \text{ of } \frac{1}{2})\ 3 - (\frac{1}{5} \text{ of } \frac{1}{2}) \cdot \frac{3}{8}} = 1 \text{ year } 3 \text{ months } 12\tfrac{6}{7} \text{ days}$$

The details of the question and the solution are not clear.

M 5.

M 5. 34 recto. i. The problem is ; Eleven birds feed on *prasriti* (handful) of corn; how many can feed on 8 Kharis of corn ? It ends "Say, O' friend, say what are the Khagas, O' Sundari.

If this be correct, the same Sundari, "beautiful one" is used in exactly the same way as Lilavati is used by Bhaskara.

The solution is 1 pra° : 11 Kha° :: 8 kha°. : 63,360 khagas which would make 720 prasriti 1 khari; but there are many elements of doubt and the application of *esha bahu pramanam* to this particular problem is not clear.

ii. By certain persons one kala plus one pada and one yava are given in gold daily at the shrine of Shulin. What would be the amount of the gift in five years, five months and fifteen days... I desire to know that.

Solution—1 day : 1 yo° + 1 ka° + 1 pa° :: 5 years, 5 months 15 days : x.

or 1 day : 30 pa° :: 1,965 day ; $\frac{1965 \times 30}{192 \times 25}$ tola

= 12 tola° + 3 dha° + $1\frac{1}{2}$ am°.

Sec Part I, Sec. III (Kaye for measures of weight)

ii. The problem is about a diamond weighing $1\frac{1}{2}$ mashaka, and obtained for ? 55 satera.

The statement means $1\frac{1}{2}$ ku° + $\frac{1}{2}$ ma°, 55 sa°, and indicates that 128 ma° = 1 ku° = 1 and that 40 si° = 1 ma°. See part III (*Kaye*).

M 6.

M 6. 37 recto. i. The question may be roughly restored : the Sun (*Surya*) traverses 500,000,000 yojanas in a day. State with certainty the amount of the journey of the Sun (*Divakara*) in a ghatika.

60 ghatika = 1 day = 30 muhurta.

The statement means 30 mu° : 500,000,000 yo° : 1 gha° : 8,3, 333, 333$\frac{1}{3}$ yo° and it indicates that 2 ghatika = 1 muhurta (= $\frac{1}{30}$ of a day). The origin of the length of the daily journey of the Sun, namely 500,000,000 yojanas, is not known.

ii. The chariot of the Sun (Bhanu) is surrounded by the groups of Gods, great snakes, Siddhas and Vidyadharas. In a day and night, its journey is said to be half a hundred kotis. Tell me, O best of calculators, how much in one muhurta ?

1 day = 30 mu°

30 mu° : 500,000,000 : : 2 gha° : 16,666, 666$\frac{2}{3}$ yo°.

57 verso. i. The remnant of a problem possibly related to the daily motion of Jupiter, which according to the *Surya Siddhanta* amounted to very nearly 5 minutes of arc (*lipta*).

ii. If *Bhanuja* (Saturn) moves through a sign in two and a half years, state, O knower of the truth, what will its motion in a a solar day be equal to (1 sign = 30° = 108000″) ? The solution is 2$\frac{1}{2}$ years : 1 sign : : *d* degree : *x*

$$x = \frac{1 \text{ sign} \times \frac{1}{360} \text{ degrees}}{2\frac{1}{2} \text{ years}}$$

$$= \frac{30 \times 60 \times 60 \times 2}{5 \times 360} :: \frac{108{,}000}{900} = 120'' = 2 \text{ minutes.}$$

of arc (not 2 seconds as stated in the text, where *vilipta* appears to have been written by mistake for *lipta*) The terms employed are all orthodox except perhaps *vasara* for solar day, but its special use is quite intelligible.

M 7.

M 7. 47, recto. i. This appears to *Partha*, the Mahabharata hero who pierced each soldier with 16 $(1-\frac{1}{3})(1+\frac{1}{4})$ arrows and slew four divisions of the army. How many arrows did he use ?

1 si° :: 16 $(1-\frac{1}{3})(1+\frac{1}{4}$ :: 4 : x; 21, 870: 2, 624, 40)

The abbreviation si° = ?; a° = anikini. See Part I. See 52 *Kaye*).

There is a very similar example about Partha in the *Lilavati* (67) which has already been quoted (Part I, 47 recto).

i. Apparently, 3 chamus = 1 pritana; 3 pritanas = 1 anikini and 10 anikinis = 1 akshauhini. The statement means that a *Patti* consists of ratha + 1 gaja + 5 nara + 3 turaga i. e., 1 elephant + 5 foot soldiers + 3 horsemen) and that an akshauhini contains 3.710 of each of these, namely—

1×3^7	chariots =	21,870 chariots (ratha).
10×3^7	elephants =	21,870 elephants (gaja).
$5 \times 10 \times 3^7$	footmen =	109,350 footmen (nara).
$3 \times 10 \times 3^7$	horsemen =	65,610 horsemen (haya).

Total 218, 700

Albiruni gives the following scheme :—

Each	akshauhini	has	10 anikini
"	anikini	"	3 chamu.
"	chamu	"	3 pritana.
"	pritana	"	3 vahini.
"	vahini	"	3 guna.
"	guna	"	3 gulma.
"	gulma	"	3 senamukha.
"	senamukha	"	3 patti.
"	patti	"	1 ratha.

M 10.

M 10. 55 verso. If tola costs thirty-five drammas (dr^o), what will be the price of one and a half tolas, one and a half mashakas and one and a half andikas and one and a half yavas.

1 tola = 12 mashakas; 1 mashaha = 4 andi = 16 yava.

$$1\tfrac{1}{2}\text{ to}^o + 1\tfrac{1}{2}\text{ ma}^o + 1\tfrac{1}{2}\text{ an}^o + 1\tfrac{1}{2}\text{ ya}^o = 319\tfrac{1}{2}\text{ ya}^o.$$

Statement - (i) $1\text{ to}^o : 35\text{ dr}^o :: 1\tfrac{1}{2}\text{ to}^o + 1\tfrac{1}{2}\text{ ma}^o + 1\tfrac{1}{2}\text{ an}^o + 1\tfrac{1}{2}\text{ ya}^o : 58\frac{76\frac{1}{2}}{192}$

or $$1 : 35 :: 319\tfrac{1}{2}/192 : 58\frac{76\frac{1}{2}}{192}\text{ dr}^o$$

M 11.

Example—One produces ten and a half in two and one-third days. For the sake of religion, he gives thirteen and one-third in three and one-eight days; he offers for Vasudeva one-quarter less than thirteen in eight and a half days. Desiring reward in a future world he gives i.e. $12\frac{3}{4}$ to Brahmanas for food, one-third in three and one-fifth days.. .... (two and a quarter in five days.. .. ... (con'd. on M 12).

M 12.

M 2. 43 recto. and also twelve and a half in thirty-three and one-third days for the best wine for the consumption of merchants. In the treasure house is stored twelve hundred. Say, O Pandit, how long can this expenditure continue. The statement means—

$$\text{Daily income} = \frac{10\frac{1}{2}}{2\frac{1}{3}} = \frac{9}{2}.$$

$$\text{Daily expenditure} = \frac{13\frac{1}{3}}{3\frac{1}{8}} + \frac{12\frac{3}{4}}{8\frac{1}{2}} + \frac{1\frac{1}{3}}{3\frac{1}{5}} + \frac{\frac{1}{2}}{1\frac{1}{2}} + \frac{1}{3\frac{1}{3}} + \frac{2\frac{1}{4}}{5} + \frac{12\frac{1}{2}}{33\frac{1}{3}} = \frac{1807}{240}$$

The daily loss is, therefore, $\frac{1807}{240}-\frac{9}{2}=\frac{727}{240}$ and

$$\frac{727}{240} : 1 :: 1200 : x$$

where x the period.

$$x=\frac{1200}{\frac{727}{240}}\times\frac{1}{360}\text{ years}=\frac{800}{727}\text{ years.}$$

M 14.

50 recto. At the top of this page is the remnant of a problem. too broken up to make out. The rest of the page is devoted to what appears to be a colophon. This is not all clear but what remains seems to state that the work was written by a certain Brahman, a prince of calculators, the son of Chhajaka. It also refers to the importance of the science of calculation which, it is said, we owe to *Shiva*

The Bakhshali Text

Transliterated in Devanagari Script

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A 1.

(40a recto, 39a recto, 38b recto, 39b recto 40b verso, 39a verso 38b verso)

यत्र य. ग...भागं चैव कारयेत् क्षैत्रवैपुल्य...पृष्ठ शत-द्वयं चैवोचरे गाता-कतः वैपुल्याद्वि...श द्वादश नृशकस्तथा। सप्तपञ्च भवेत् चानं भक्ति स्थाने...र... धा सप्त पञ्चानां त्रि-द्विमेक ф प्रकल्पितम्। तस्य वाहस्य किं क...तत्र गग। क्षेत्रस्य स्थापनं क्रियते।

क्षेत्रम् १००

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...चैव तत् फल...गुणिता जाता |६२१०| एष वाहस्य काण्ड प्रमाणं ...शके मूल्यं कर्त्तव्यम्।

अधच्छेदं चतुष्षष्ठि...ल। सुथद्वि त्रिशभि मण्डलकौ टल्लिका एशच्छेदं भवति ...यथेच्छ... कार्या। शुथटल क्रिया उदाहरणं...टलस्य...मेकं तद्वा षष्टि शतानी दशाधीकानां किं मूल्यं : टल्ल...तले अ०

१...ल १ मूल्ये १	६२१० मणले आ इम्	फ...

A 2.

(39 recto, 40 recto, 38 recto. 39 verso, 40 verso, 38 verso)

...क्ष...दशा चतुर्दशा तृतीयस्य चतुर्थस्य...भागास्तस्यैव पञ्च...भागा विशषष दशगुणा। सप्तम क्ष...ज्ञ यं शतम्। सर्वे मिश्रापि दृष्टंच शतानि ......

...धनं १......१०॥ एष एकैक भागागुणिता जा ... ६०।१८०।२००।३०० एवं धनं १२०० प्रत्यय त्रैराशिकेन...००...१२००

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—	फ० १६—
—	फ० १८०
२०	फ० २००

A 3.

(40 recto, 39 recto, 38 recto, 40 verso, 39 verso, 38 verso)

ब्धाम्बु पयसो घटः एक मिश्रिकृतक...

करणम् । हव्य तुल्यं विनिक्षिप्यः

	५	६
४	५	६
	५	६

कुरु प्रक्षेपकं तत प्रक्ष

स्थाप्य

$\frac{४}{१५}$	$\frac{५}{१५}$	$\frac{६}{१५}$
$\frac{४}{१५}$	$\frac{५}{१५}$	$\frac{६}{१५}$
$\frac{४}{१५}$	$\frac{५}{१५}$	$\frac{६}{१५}$

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...१२० ।। २२५७...णम्...त् शेषे क्षेप १६ अनेनात्र भाग

| $\frac{३२}{१६}$ | लब्ध | २ | | $\frac{४०}{१२}$ | फ... फ१२० लब्धेर्भाग | $\frac{२८}{२}$ | जाता १४ लब्धक्षेपं...द्‌० ६० प्रक्षेप युक्ति ३० विभक्तं | $\frac{१}{३०}$ | ...णिता

जाता... १४।१८।२८। एवं ६०

A 4

(54 verso, 54 recto)

(i) '''द्विगुणञ्च न्यून च तृतीयस्य धनं भवेत् ' न्यून'''संयुतम् ।
एकविंशतिभि '''कृतो दीनारैत्तर आयतुदंसापृथग् वद: ।।
करणम् ।। यस्य पद्मं न ज्ञायते एतद् प्रथमस्य धनम् ।

(ii) '''च दत्तवान् हस्ता'''येषां | ० | ३ | २+ | '''

धनं''' '				
	१	२	४	८
	१	१	१	१

याता । तयोर्योगवियो'''कृतां राशय:

२	१	२+	६+	दृ० ८२
१	१	१	१	
१	२	४	८	
१	१	१	१	

'''भाज्या हित्वेति । तत्र उत्तर राशि''' उत्तरं ऋणं जातं
(b) सूत्रम् ।।'''(c) जातं ७६ एव प्रथमस्य'''

A 5

(35 recto, 35 verso)

'''कस्मात् कारणा । तयोर्योगवियोगस्याधियोगम् भाजिता पुरुष १२ अंश

भक्त्वा धनं | ६/१५ | '' पद्धय ' सहितम् ।।

'''मूलेन | १/१/२ | एत द्विगुणं | ३ | द्वियुतं'' यस्य धनम् ।

तदेव सार्धं | ३ | अस्यार्धं | १/१/२ | युतं न्यास

A 6.

(51 recto, 51 verso, 35 verso)

भाजितो हित्वा । तत्रोत्तरा १ । १ । युतं | २ | | १/४ ३/२ ३/१ | ६ | ...

एषा...ईथ भाजिता । पुरुषः | १/४ | ३/२ | ३/१ | एषां सदृशे | ४/१६ | धनं | १६/१ | अनेन

गुणितं जातं | ४ | एष प्रथमस्य धनं ... द्विगुण | १२ | द्वियुतं | १४ | एत

द्वितीयस्य ... गुणं | २१ | द्विगुणं | ४२ | त्यूनं | ३६ | एषः न्यासः प्रत्य ...

दशमग्र·वृन्दानां चतुर्दश एकोनचत्वारिंश। तत्...पादार्धं त्रिभागा...४ | १/४ | फ० ४ |

| एवं दीं० १ | एष प्रश्न एतैर्...

उदा ॥... | ६ | ...यांग १११ शेषा φ पुरुष

भाजिता पुरुषाः १/४ | १/६ | १/५ एषां सदृशे युतिं कृत्वा | ३७/६० | भाजिता | ६०/३७ | ...

एष गवाश्व महिषी प्रत्यैक शालेषु भाग...

१ शा०	१८० गा० / १	१/४	फलं ४५...	
१ शा० / १	१८० अश्वा० / १	१/६	फलं ३० ४ +	२६
१ शाला / १	१८० महि० / १	१/५	फलं ३६ ५ +	६

A, 7

(51 recto, 51 verso)

| १/१ | १/५ | १/१ | १/१ |

...। १ । २ । ३ । एषां युत | ६/१ | ... | ४८ | शेषा φ पुरुष स ४ ॥

अनेन भाजितार्लब्धा.. स्य भवति । १२ । १३ । १४ । १५ । एकत्वं ५४ ॥

(ii) उदा० ॥ कश्चिद् राजा ददे दानं सप्तपञ्चाशकं बुध ।

पञ्च······प्रवक्ष्याम्यनुपूर्वशः ॥
द्विगुण-द्विगुणं चैव रूपरूपोत्तरे···
···प्रथमे प्राप्तं किं प्राप्तमपरे जने ॥

०	१	२
१	१	५

द्द० ३२८
१

१	३	९	२७	८१

करणम् । उत्तर···तत्रोत्तर राशिनां योग ८७ एष धना दृश्या शोधनीया जाता २४२···। पुरुष । १ । ३ । ९ । २७ । ८१ । योग १२१ अनेन···जाता | २ | एष द्वौ प्रथमस्य धनम् ॥

२ । ६ । १८ । ५४ । १६२ । उत्तरराशीसंयुतंजातं

२	१५	४८	१४७	४४४ एषां···
१	२	२	२	२

A 8.

(52 recto., 52 verso.)

(i)···५७···त्वेदं जातं ३ अनेन चालिश गुणयं जात। १२०···स्ववंशुराणाम्"

प्रत्यय त्रैराशिकेन

१	१२०	१	१२	···
१	१	१०	१	
१ वं	१२०	१	१५	···
१	१	८	१	
१ वं	१२०	१	३०	···
	१	४	१	

(ii) उदा० ॥ धना स्वमर्धों संशोद्ध्य···चतुरीयकम् ।
तत् शेषा पञ्चमो भागो···शतत्रयम् ।

अशीत्याधिकं धनं चैव किमाद्यं प्रथमं धनम् ॥

···अस्य द्वयानां शतानां पाद···र्धाम् ॥

शतं भवति १५० अत्रापि पञ्च भाग ३० ॥ एवं···

१ २	१ ४ १ २ +	१ ५ १ ४ + १ २ +	फ०पिण्ड २८० ॥

पञ्चमी जाती करणं कृत···२८० । अंशयुति २८ भक्तं ४० धनु० २८०
४० २८
गुणितं जातं ४०० एष फलं भवति ॥

A 9.

(29 recto. 29 verso)

··· ···विर्निर्दिशेत् ॥

उदाहरण ॥ धना··· ···

स्य द्वितीय योन्मिश्रं धनं तत्र त्रयोदशः
द्वितीय तृतीय योन्मि···चतुर्दश
आद्य तृतीययोन्मिश्रं धनं पञ्चदश स्मृतः
एकैकस्य धनं···च्छिचे कथ्यतां मम:

१३ १	१४ १	१५ १	

···प्रथमन्यस्य तत्रेच्छा पञ्चः | ५ | तत् प्रथम···

···| १३
१ | १४
१ | १५
१ तदादीश्शोधयेत्क्रमातादि···एत

चतुर्दशभि शोध्य शेषं ६ एतत् पञ्च
··· ··· द्वितीय योन्मिश्रं धनम्

द्वितीय तृतीय योग्मिश्रं धनं सप्तदश स्मृत:
तृतीयाश्चतुर्बयो··· ···
चतुः पञ्चक मिश्रं तु धनं एकोनविंशति ।
प्रथम तत्र च
एकैकस्य धनं किं स्याद् वेच्छि···

१६	१७	१८	१९	२०
१	१	१	१	१

करणम् ॥ इच्छा ··दानीशोधयेत् क्रमात् तत्रादि १६ शुद्···तृतीयायं शोध्य ७ चतुर्थायं शोध्य १२ पं०··· ···

A 10.

(27 verso, 29 verso, 27 recto, 29 recto)

······मस्य धनम् । एषामनुक्रमेण पूर्वोक्त······

९ प्र०	७ द्वि०	१०त्रि०	८ च०	११ पं०
७ द्वि०	१०त्रि०	८ च०	११पं०	९ प्र०

युतं जातं प्रत्यैक···१६। १७। १८। १९। २०···एवं सर्वत्र कारयेत्

करणम् । पृथक् रूपं विनिक्षिप्य । पृथक् रूपं क्षिप्तं जातं·· ···भ्यासो तत्र गुण | ३ | ४ | अभ्यासं | १२ | रूपहीनं १······अभागा चतु♭ पञ्च ४५ । अत्र क्षिप्तं जातं | १५ | १६ | एष त्रिगुण······ता मूल······नि चतु♭ पञ्च | ५ | ४ | एप··· सूत्रं ॥ गुणो···क धनम् ॥ गुणन्यासो रूपहीनं लब्धं रू······
[♭ = उपध्मानीय]

A 11

(1 recto, 1 verso)

(i) भ्यास | ··· | रू······चतुर्गुणं पञ्चगुणं हस्तगतं धनं ज···पञ्चगुणं २५।।
| १ |

नवम सूत्रं ९ ❀

(ii) सूत्रम्। गुणौ पृथग् रूपयुतौ याचना युक्ति संगुणौः ॥

गुणनेण।गुणे···रूपहीनेन भाजितौ ।

विपरीत याचना क्षिप्तौ गुणसास्योरयं विधिः ॥

एवं सूत्रम् ॥ द्वितीय पत्रे विवरितास्ति ।

दशम सूत्रम् १० ॥ ❀

(iii) सूत्रम् । ॥ अंशां विशोध्यच्छेदेभ्य-कुर्यातत्परिवर्त्तनम् ।

···ास्यं तत प्रोज्झ्य धनान्विश विनिर्दिशेत् ॥

(iv) उदा० ॥ पञ्चानां वणिजामध्ये विक्रीयते किलः ।

तत्रोक्ता मणि-विक्रीता मणि-मूल्यं कियद् भवेत् ।

··· ··· दम् ।

अर्ध त्रि-भाग पादांशं पञ्चभाग षडंश च ॥

······स्यं ततो प्रोज्झ्यः सदृशं क्रियते···जाता

१२०	९०	८०	७५	७५
६०	६०	६०	६०	६०

तत्र प्रोज्झ्य जातं १२०। ९०।

८०। ७५। ७२ एषां योग कृते जाता ४३७ अतो···शेषं ३७७ एष मणि मूल्यम् ।
चतुर्णां संक सर्वस्वम् ॥

प्रथमस्य संकार्धं···९०। ८०। ७५। ७२ चतुर्णां योग ३१७ प्रथमार्धेण षष्टिभि-र्युतं ३७७ प्रथमस्य धनं प्रथम धनं ।

तृतीय चतुर्थ पञ्चमस्य धनं सर्वस्वं ३४७ द्वितीय त्रिभागं ३० एष युतं ३७७ एष द्वितीयस्य धनं भवति ॥

पुन प्रथम द्वितीय चतुर्थ पञ्चम···सर्वस्वं ३५७ तृतीयस्य पादं २० एष युतं ३७७ एष तृतीयस्य धनं भवति ॥

पुनरपि प्रथम-द्वितीय-तृतीय-पञ्चमस्य ३६२ चतुर्थस्य पञ्चोभाग १५ एष युतं ३७७ एष चतुर्थस्य धनं भवति ।'

A 12

(2 recto, 2 verso)

(i) स्य धनं भवति ॥

अथ प्रथ···त्यं षष्टि शेषं ३७७ ॥

अथ द्वितीयस्य	१२०	एवं ३७७ द्वितीयस्य भवति ॥
	३०	
	८०	
	७५	
	७२	

अथ तृतीयस्य क्रियते	१२०	एवं ३७७ तृतीयस्य धनं भवति ॥
	९०	
	२०	
	७५	
	७२	

चतुर्थस्य क्रियते	१२०	एवं ३७७ चतुर्थस्य धनं भवति ।
	९०	
	८०	
	१५	
	७२	

पञ्चमस्य क्रियते । स्थापनं	१२०	एवं पञ्चमस्य ३७७ ।
	९०	
	८०	
	७५	
	१२	

एष मणिमूल्यं प्र······

(ii) उदा० ॥ अन्योऽन्य विदित विभवं···वणिक द्वयम् ।

त्रि······ दलं तथा······

७+	३+	५+
१२	१२	६
१२	१२	६

अंशं विशोद्ध्य विशोधयेत् ऋणं स्थितम् । एषं···क्रियते

९	७	११	कुर्यात् परिवर्त्तनं	१२	४	६	च्छेदे
१२	४	६		९	७	११	

···जातमस्य	९२४	८३६	७९८	प्रोज्झ्य जाता
	१४६३	१४६३	१४६३	

| ९२४ | ८३६ | ७९८ | एषां युतिं क्रियते···जाता । २५५८ । च्छेद प्रोज्झ्यं १०९५ एतन्मणि मूल्यम्'

A 13

(3 verso, 3 recto)

उदा० ॥···द्वितीयस्य हयान्नव: ऊष्ट्रादश तृतीयस्य···प्रदत्तं च परस्परं पृथग्धनं तु वणिजां मूल्यं वा प्राणिनां पृथक् यदि···वक्तं ततोमेच्छिन्धि संशयः ॥

७ अ०	९ ह०	ऊ० १०
१	१	१

वणिज्जका ३ देयं···वणिक् पिण्ड हृतं । पिण्ड ७।९।१०। देयं ३ शुद्ध शेषं ४।६।७ तत शेषं परस्पर कृतं गुणित जातं ।

।१६८।१६८।१६८। स्वशेषेन तु विभक्तं

१६८	१६८	१६८
४	६	७

लब्धं ४२।२८।२४। एष प्रत्यैक मूल्यं एकैकस्य···गुणिता जातानि अश्वै हयै ऊष्ट्रेभ्यः २९४।२५२।२४० एकैकस्य···जाता २६२।२६२।२६२। एतेस्समधना जा

(i) त्वास्समधना जाता प्रस्तमूल्यं तदुच्यताम्

४ य०	५ गो०	६ शा०
१	१	१

एवं प्रस्त मूल्यं २।३।: दत्तैस्समधना जाता १७।१७।१७ ······ सम-सतं १३ ❀

(ii) सूत्रम् ॥ एकयुतानां संख्या द्वि···हीना च ।

एवं तावत् कार्यं यावत् पुरुषं समा भवति ॥

सप्तम पत्रेभिलिखित स्थित

चतुर्दशं सूत्रं १४

(iii) सूत्रम् ॥ गतिस्यैव विशेषञ्च विभक्तं पूर्वं गन्तुनाः तेनैव कालं भवति स्य केन तु ॥

(iv) उदा० ॥ अद्ध्यर्ध योजन गते शत··· ··· ··· ···

B 1.

(8 recto, 8 verso)

(i) ❀ सूत्रम् ॥ द्विगुणं प्रभवं शुद्धा द्विगुणा नियथं तथा ।
उत्तरेण भजेच्छेपं लब्धं रूपं विनिर्दिशेत् ॥

(ii) उदा० ॥ वर्त्तते भूतक X कश्चितत्रैको दश माशकम् ।*
स्यहं कुरुते तत्र कर्मं भट्टिकमानवः
द्वितीयं क्रियते कर्मं द्व्यादि तृतीयरुत्तरम् ।
पदं तत्र तु भवति केन कालेन सास्यताम् ॥

| आ० २ | उ० ३ | पा ० | प्रति० १० |
| १ | १ | ० | १ |

द्विगुणं प्रभवं शुद्धा प्रभवं | २ | द्विगुणं | ४ | नियत पुन द्वि···

···१६ [उत्तरार्धेन भाजयेत्] उत्तरम् ··

सूत्रम् ॥ हयोर्विभज्य गन्तव्यमतो भाग० गन्तत ❀ एकाश्चगमनशेय युताःसं-गुण्य ··

उदा० ॥ नियो रथोश्वैर्दशभिर्युज्यते हय पञ्चकं । गन्तव्यं योजनशतं··· किमुप्-

भवेत् | ह० १० | हयलग्न रथस्य | ५ | गन्तव्यं योजन १०० |
| १ | | १ | | १ |

हयोर्विभज्य गन्तव्यं तत्र हया | १० | गन्तव्यं यो० | १०० | अतो भाग हृते लब्ध | १० | तत्र युक्ताश्व | ५ | एतैस्संगुण्य परियोग जातं योजनान्यैकोश्वस्य ।
प्रत्ययः पञ्चभिस्शतं संगुण्य जातं ·· क्रियते । यदि द ··योजना ɸ पञ्च···

* X = जिह्वामूलीय, ɸ = उपध्मानीय

B 2

(9 recto, 9 verso)

उदा. ।। ...तत्समाप्तं द्विजन्मभि ।

तत्पुनस्ते समं भक्त्वा दश ...स्समाप्तवान् ।

संख्याय X कतिम्माचक्षु कति विप्रा X कति प्रष्टम् ।।

आ० १	उ० १	प० ०	लब्धं १०
१	१	१	१

करणम् ।। लब्धं द्विगुणितं कृत्वा तत्र लब्धं । १० । द्विगुणं । २० । तथाचूनं १८ / १

उत्तरेण विभाजितमत्रोत्तरं । १ । अनेन भक्त्वा जातं तदेष रूपाधिकं | १९ / १ | अयं प्रश्ना प्राहुणा एकोर्नबिंशति

स्थाप ...प्रत्ययं आ० १	उ० १	प० १९	रूपोणाकरणेन फलं १९
१	१	१	१

यो० ६	शयो० १	यो० ७०	गन्तव्यं
१	१	१	

अलब्धे संयोग | ७ / १ | विभक्तं | १ / ७ | गन्तव्येन गुणिता जातान् लब्ध । १ ।

द्विगुणं । २० । एषाल्पस्य: ।। अथ ...अयं कालोज्ञेय: अनेन कालेनषुषट्-येन..।।नि गन्तव्यम् । —यमेक योजनिकस्य समागमो भवति ।।

तद्यथा त्रैराशिकेन प्रत्यय । यद्यं कस्य षट्योजना तर्दाविंशानां किं अथ

१	६ यो०	२०	फ० १२०
१	१	१	१

सप्तति शोद्ध्य शेष अत्रस्सप्तति | ७० | आगतपञ्चाश | ५० | अत्र...

१ दि०	१ यो०	२० दि.	फ० यो० २०
१	५	१	१

B 3.

[7 verso.]

[i] ………

आ० ३	उ० ४	प० ०	नित्यदत्त ७
१	१	१	१

आदि विशोद्ध्य…आदि | ३ | नियतं | ७ | विशोद्ध्य | ४ | उत्तरार्धेन भाजितम् । उत्तरं | ४ | अनेन भाजितं | ४/२ | जातं | २ | लब्धंसरूप । एप रूपाधिकं | ३ | एष काल…

आ० ३	उ० ४	प० ३	रूपोणा करणेन फलं रू० २१ ॥
१	१	१	

द्वितीयस्य त्रैराशिकेन…	१ दि०	७	३ दि०	फ० रू० २१ ॥
	१	१	१	

एषस्समधना जाता ॥

(ii) उदा० ॥ आद्येक उत्तरद्वयं द्वितीय-पञ्च-प्रत्यहम् ।
केन कालेन समतां वद मे गणकोत्तम ॥

आ० १	उ० २	प० ०	नियतनित्यं ५
१	१	१	१

आदि विशोद्ध्या……

B 4.

(4 recto 4 verso)

(i)…योजन पञ्चकम् । सप्तदिनानि तस्यैव गतस्य । परत द्वितीय मय योजनैक गतके…तां

१ दि० ५ यो०	दिन ७ गतस्य गतयोजन ३५	द्वि० १ दि० ६ यो०
१ १	१ १	१ १

गतिस्यैव विशेषञ्च…यते । गति ५ । ६ । विशेषं । ४ । विभक्तं | १/४ | पूर्वगत

३५ एष पादेरगुणितं $\frac{३५}{४}$ ···भिर् दिनै सम गती भवन्ति नव योजनम् ॥

प्रत्यय त्रैराशिकेन				
	१	योः	३५ ···	
	१ दि०	१	४	
	१ दि०	६ योग०	३५	फ०···
	१	१	४	

उदा० ॥ अष्टादश योजना एकेन दिने याति ।

तस्याष्ट दिना गनस्य ।

द्वितीय पञ्च विंशे योजना दिने याति ।

केन कालेन सास्यताम् ॥

एवमेकादशम पत्रेभिलिखित पूर्वेऽपि ॥

पञ्चदशम सूत्रम् १५ ❁

(iii) सूत्रम् ॥ आद्योर्विशेष कर्त्तव्यमुत्तरस्य विशेषतः ।

विभक्तमुत्तरे···

उत्तरं | $\frac{२}{१}$ | विभक्तम् | $\frac{१}{२}$ | आदि शेष $\frac{२}{१}$ | जाता | $\frac{१}{१}$ | द्विगुणं | २ | रूप संयुतं | ३ | एष··· संकलिते प्रत्यया पदंहीना उभये स्थापितव्या रूपोणा-करणं फलं | $\frac{२१}{२१७}$ | द्वि कि प्रभूतेऽपि लिखिते ॥ षोडशम सूत्रं १७ (सूत्रे भ्रान्तिमस्ति)

(ii) सूत्रम् ॥ आद्योर्विशेष द्विगुणं च य शुद्धिर्विभाजितं ❁ रूपाधिकं तथा

कालं गतिसास्यं तदाभवेत् ॥

(iii) उदा० ॥ द्वयादि त्रिचयश्चैव द्वितीय त्यादिकोत्तरः ।

द्वयोच भवते पन्था केन कालेन सास्यताम् ॥

स्थापनं क्रियते ॥

आ० $\frac{२}{१}$	उ० $\frac{३}{१}$	प० $\frac{०}{१}$	द्वि०	आ० $\frac{३}{१}$	उ० $\frac{२}{१}$	प० $\frac{०}{१}$

करणम् । आद्योर्विशेष···

C 1.

(5 recto, 5 verso)

आ० ५ / १	उ० ६ / १	प० ० / १	घ० ० / १
आ० १० / १	उ० ३ / १	प० ० / १	घ० ० / १

करणम् । आद्योर्विशेषम् । आदि ''चय शुद्धि चयं ६।३। शुद्धि | ३ / १ | आदि-शेष ५ द्विगुणं । १० । उत्तर विशेष ३

विभक्तं | १० / ३ | सरूपं | १३ / ३ | अनेन का'''समधना भवन्ति।।

प्रत्ययम् । रूपोणा-करणेन फलं | ६५ / द्वि० ६५ | एष पदं'''अष्टादशम सूत्रं १८ ।।

(ii) (सूत्रम्) ।। दिन गमनमादि रहितं द्विगुणं तच्चोत्तरेण ❀ संयुतम् ।
प्रतिनिहिताऽत्मगुणं ज्ञेयं क्षेप संज्ञको राशि ।
अष्टोत्तर गुणिते क्षेप संज्ञको दावा मूलं
प्रतिनिहित युतं द्विगुणोत्तर भाजितं

''' ''' ''' '''हतं ३०'''दिन गमनमादि रहितं दिन गमन योजन ф पञ्च | ५ | आदि । ३ । रहितं जातं | २ | द्विगुणं | ४ | तच्चोत्तरेण संयुतं | ८ | '''आत्मगुणं । ६४ | एष क्षेप संज्ञको राशि अष्टोत्तर संगु''' लब्ध राशि | ३० | अष्टगुणं | २४० | उत्तरेण गुणमुत्तरं | ४ | '' गुणितं जातं | ९६० | क्षेप संज्ञको दत्वा । तत्र क्षेप संज्ञ''' ६४ | युतं जातं | १०२४ | अस्य मूलं | ३२ | प्रति निहित''' | ८ | युतं जातं | ४० | उ''' . . .

C 2.

(6 recto, 6 verso)

(8) शिके प्रत्यय | १ / १ | ५ / १ | ५ / १ | फलं''' अनेनस्सह ५५ एष समाब्धानम् ।।

३

(ii) उद० ॥ आदि पञ्चमुत्तरं त्रीणि नरो योजन गम्यते ।
द्वितीय प्रति दिनं स्सप्त गतस्य दिन पञ्चकम् ।
केन कालेन समतां कथ्यतां गणकोत्तम ॥

आ० ५/१	उ० ३/१	प० ०/१	प्रति गति ७/१	दिन ५/१

पञ्च दिन ग···योजनिकं योजन | ३५ | करणम् । दिन गमनमादि रहितं तत्र दिन गमनं | ७ | आदिरहितम् । आदि | ५ | रहितं ··

(i) ···अनेन गुणितं जातं | ८४० | संज्ञको दत्वा तत्र क्षेप राशि | ४९ दत्वा जातं | ८८९ | ·· दान ददाति समम् । करणो क्रियते ।

(ii) सूत्रम् ॥ अक्रियते श्लिष्ट कृत्यूना शेषच्छेदो द्विसंगुणः तद् वर्गः दल
संश्लिष्ठ हृति । शुद्धि कृति क्षयः ।
अनेन सूत्रेण श्लिष्ठमूलमानय स्वमतिमा···

(ii) लब्धं मूलं | २९ / ४८ / ५८ | प्रतिनिहितं | ७ | अनेनयुतं | ३६ / ४८ / ५८ |

··· | २१३६ / ५८ | द्विगुणोत्तर भाजितम् । ततो······

G 3.

[7 recto]

६ १ ८ ६० १६च० ६० ३३ लि० ६० ६ वि० ६० शे० ६ २९	|४४७/२९| दलिता |४४७/५८| ···सास्येयुतं |७३७/५८ पदघ्ना । तत्र पदं |१७८/२९| अनेन गुणितं जातं |६५५९३/४८१| श्लि···त्य शेषं क्रियते । |६५५६९/८४१| भागे हृते प्रत्ययं त्रैराशिकेन |१/१| ७यो०/१ | १७८ फलं/२९

योजन ४२ शे	२८ २९	नियतं तेन···	७७	···एकोन विंशतिम सूत्रम् १९ ॥

C 4.

(65 verso)

आ० १ १	उ० १ १	प० ० १	६० १

करणम् ॥ अष्टोत्तरघ्ने गणिते । अष्टघनं । ४८० । उत्तर घन ' द्विघ्न· मादि । आदि द्विगुण । २ । चयोज्झितं । च उत्तरम् । अतो उत्तरं पातयित्वा एकं भवति । १ । व ' निक्षिप्य धनस्य । ४८१ । मूलं श्लिष्ठ क॰णा

२१ ४० ४२	···वंशं	८८२ ४० ४२	शेषं

चत्वारिंश पृथक् स्थाप्य ······८८०	४०	···योज्यं	९२२ ४२	तन्मूल वर्जितम् ॥ तन्मूलं

C 5.

[56 verso, 56 recto]

८८० ८४	९६४ १६८	गुणित जातं	८४८३२० १४११२	चत्वारिंश पृथक् स्थानां वर्गं ।१६००।

एष उपरापात्य शेषं	८४६७२० १४११२	वर्ग्य जातं	६०

···	२१ २० २१	तेषां वर्गः तस्यात्···

अकृते श्लिष्ठ कृत्यूनात् शेषच्छेदो द्वि संगुणम् ।

तद् वर्ग दल संश्लिष्ठः हृति-शुद्धि-कृति-क्षयः ॥

शेषच्छेदो त्रि—संगुण कृ···

२१				२१ भा
२०	४००	दल १	संश्लिष्ठः	२० +
२१	४४१	२		२१

शेषं पात्य···त्वा भाजित···अधमुपरे उपरं गुणितव्यं वर्गं या वर्जये···म

··· | ४२५०४२ / १९३६२ | ४०० / १९३६२ | शेषं | ४२४६४२ / १९३६२ |

C 6.

[64 recto]

| ४०५२८० / ३८७२४ | ४४४००४ / ३८७२४ | ···अर्धं कर्त्तव्यं···

| ४०५२८० / ३८७२४ | ४४४००४ / ७७४४८ | संगुण्य जाम् । अ··· ह्वरा हरेषु गुण्

··· | १७९९४५९४११२० / २९९९९९६३५२ | अस्य ऊर्धाङ्क·· १६००००+

···शेषच्छेदो द्विसंगुणं | ६ / ५ / १२ | शेषं १ज्चकं पृथक्

··· अंशास्वंशं | ७७ / १२ | तन्मूल वर्जितं । तन्मूलं द्विगुणोत्तर संभक्तं | ६५ / २४ |

एष पदम् ॥···नगनं | आं० १ / १ | उ० १ / १ | प० ६५ / २४ | रूपोणा | ४१ / २४ | ···दलिता

| ४१ / ४८ | आदि संयुतं | ८९ / ४८ | ··

C 7.

(57 verso 57 recto)

···अष्टोत्तर—घ्ने गुणिते ४० द्विघ्नमादि च...निक्षिप्य | ४१ | मूलं | ६ / ५ / ६ |

शेषच्छेदो द्विसंगुणि...

शुद्धः तस्मात् अकृते श्लिष्ठ कृत्यूना शेषच्छेदो द्विसंगुणः ।

तद् वर्ग दल संश्लिष्ठः हृति-शुद्धि-कृति-क्षयः ॥

अकृते श्लिष्ठ ··· तदाद्वि संगुणकृत

६	तद् वर्गतं	६		
५		५	२५	दल...
१२	१२	१२	१४४	

२५ / १८४८ ··· | ११८३३ / १८४८ | हृ ।१८४८। कृति क्षय कृतः एष मूलम् ॥ तन्मूल...

मूलमेकं १ एष सदृशे पातित जात | ६६८५ / १८४८ | ...सम्भक्तमुतरं द्विगुणं २ अनेन भक्त॥

| ६६८५ / ६६६ | एष पञ्चकस्य पदम् ॥ अस्य प्र—

सूत्रम् ॥ एको राशि द्विधा स्थाप्य ऊपसे—

C 8.

(45 recto, 45 verso)

(i) | १०२२५ / ३२८०० | दलिता | १०२२५ / ६५६०० | ··· आदियुतः | १०८६२५ / ६५६०० | पदघ्ना

पदसंयुता... ६४५५०४०६२५ / ३२२७५२०००० | अतो पञ्चविंश ··· उपरा: | ६४५५०४०६२५ / १२२७५ ०००० | अथ।

··· लब्धं २ एष धनम् ॥

(ii)

आ० १	उ० १	पदु ०	धनु ७०००
१	१	१	
२	२		१

···।३८४। अस्यवर्गं १४७४५६ अक्रि ···।२१७४३२७१६३६। एष सर्वे गुणि।

करणि ··· कृत्वा भाजित जातः | ११५८५ / ६७१२५० | अंशेरंशा गुणये ··· रशि वर्ग्य जातः

| ५७६ / ७६८ / ११५८   २६४६१ : / ७७७३०७५००   ५७६ / ५१५५२०००० / ७७७३०७५०० | २६४६१२ + / ७७७३०७५००

शेषं | ५७६ / ५१५२२५०८८ / ७७७३०७५०० | ——— | ४५०५६७२६७५८८ / ७७७३०७५०० |

द्वयेन मूले···

C 9.

(46 recto.)

| ४४८२४४३४५०८८ / ४६६३८४५००० | ४४३१८०५०००८८ / ४६६३८४५००० |... | १२१७६०२५००४४ / १५५४६१५००० |

दलिता ए | ११०८६५१२५०२२ / १५५४६१५००० | आदि संयुत | ११३२२७०४७५२२ / १५५४६१५००० | पदछ्ना

| ५०७५३३८३७६२७४६७४३२७१६३६ / ७२५०४८३३६४६७५०००००० | ··· करणी पात

| २१७४३२७१६३६ | पातित जाता उपरान्यास स्थाप···

| ५०७५३३८३७६२७२५० ००००००० / ७२५०४८३३६४६७५०००००० | भा···७०००

D 1

[46 verso]

| मडे ८ | मडे ६ | मडे ३ | ···का २० अपर प्रष्ट: पारा आ इ ए वीहु-जण वीहान···है ··ण । गोरे जा म च । उप्पणे सा मले अ··· धप··· ढले आपोत् डीणे आगणे वीहुजण ए हुवी ·· करणम् । तैगोदे वारे हृहि पणे हि सा··· —

D 2.

[70 (a, b, c) recto, 70 (a, b, c), verso]

तोल ५⋯⋯

⋯⋯३५ | एते भागा⋯ —
२ |

⋯⋯१७ | ११७⋯⋯
५ | ७०

⋯२ तो ० शता ७⋯पल २ तोल १⋯पल ६ ॥

उदा० ॥ समा⋯⋯णिपेशी कृतानि च । द्व चतिस्रस⋯तिस्रसमापा॥

तुलितानि त्रयोदशे ।

⋯⋯एकैकस्य सार्धंय:⋯⋯

१	१	१	द⋯
२	३	४	

प्रक्षेप-युक्त्या-फलं⋯

⋯रि⋯रि

गुण्यफल राशि⋯

⋯कर्त्त पल ८

उदा० ॥ अर्धं त्रि⋯दांशा पञ्च षष्टि नृपो ददौ ।

सेवकानान्तु दी⋯

१	१	१	दृश्य ६५	सदृ⋯⋯
२	३	४	१	

D 3

(69 recto, 69 verso)

२	२	२	दृश्य⋯अतोसदृश⋯हकम् ।
५	६	७	

उपरि मां शंतंतुला भवन्ति चत्वालिश । दूना चौराशी…एत ततंक्तुला ।

द्वि चत्वारिं …न्ति एते व्रीहिका सर्वत्र: स्थापनमस्य ……

…… ।

प्रत्यय त्रैराशिकेन | ५ आ० / १ | २ तं० / १ | २१० / १ | फ० तं० ८४

…ईयस्य क्रियते । ६…… / १

…यते राशि: | ७ / १ | २ / १ | २१० / १ | फलं

………क तं १०५…

उदा० ॥…त्रिभिर्दत्तै, त्रिगुण त्रिगुणेन तु ।

……तदुच्यताम् ॥

| १ / १ | ३ / १ | ९ / १ | दृश्य १३० / १ | प्रक्षेप | १० | ३० | ९० एकत्वं | १३० |

…वान् ॥ तं शतं त्रिभिर्दत्यै परवप्ता पवप्तृ…

| ४ / १ | ६ / १ | ९ / १ | दृ० १९० | … | ४० | ६० | ९० | एकत्वं

D 4.

(68 recto.)

… १६८ / ४ |… देश द्वात्य…पात्य जाता

शेषं २१…एकत्वं २९…द्वं २…

………बारि…

D 5.

(31 recto, 31 verso)

(i) ··त्रैराशिकेन ३ दिने | २ द्रं० | १६८ दि० | ···
 १ | १ | ११ |
 | २ | |

तीयस्य क्रियते··· | ३ दि० | २ द्रं० | १६८ दिना | फलं द्रं०
| १ | १ | ११ |
| | २ | |

| १४० | प्रथमेन दत्तं···सप्तः दत्तैस्समधना जाता ।
| ११ |

···सदृशं | ७७ | २६४ | पात्य शेषं | २१७ | द्वितीयस्य
| ११ | ११ | | ११ |

दत्त······७७ एषस्समधना जाता ॥

(ii) पुनान्यं सर्वंभा···भा ४ दिने । द्रं० १५ जीव्या ॥
 १
 ४+

द्वितीयस्य । भा ३ दिने । द्रं०···
 १
 ३

···कारणं···च्छेद संगुणे | द्रं० १ ४ तं |
| १ १ |
| २ |

| द्रं० १६ मुद्गा | ···धं युति हृत फलम् ॥ अस्य गुणाकार
| १ |
| २ |

द्वया । | २ १ | ······उपरं गुणये आदौ ताव ५ द्व···
| ६ १३ |
| २ २ | ······गुणय | २ | ···भिनय भागेषु ·· सूत्रम् ॥

D 6.

(67 verso, 67 recto.)

(i) ·· च्छेषं त द्विगुण जा···ता ।

निर्गच्छ···प्रविशमाने चत्वारि दत्त: पुन द्विगुणं जातं

शून्य हस्तं गतं तस्य किमत्र मूलधन स्यात् ।

१ २ भा० १	१ २ भा० २	२ २ भा० ३	३ २ भा० ४
१ १ १	१ १ १	१ १ १	१ १ १

४ २ भा० ५	१	···
१ १ १	४	

···	४९ १२	जाता	६१	··· सदृशं	८ ११	पुन
	८ ८		८		१ २ ८	

१६ ६१	जाता	७७	···सदृशमेकस्य	१६	युतं	७७
८ ८		८		१६		१६

जातं	९३	एष फलं भवति । ···
	१६	

प्रत्ययः	९३	१+२ २+	२+२ ३+	३+२ ४+	४+
	१६	१ १ १	१ १ १	१ १ १	१

(ii) हुण्डिका समानयन सूत्रम् ॥

दिन भक्त विशेषञ्च··· द्विगुणं क्रियते

चैव कालमेषां विनिर्दिशेत् त्रैराशिक विधानेन

···आदत्तं च पातव्यं सूक्ष्मे दत्तं च तत्समम् ॥

उदाहरणम् ॥ द्विगुण··· ···

D 7.

(28 recto.,)

तं द्विगुणं भारं···लब्धं १६ ॥ पुन क्रिया···वेत् ॥ ··· गुणये | १/११ | १/८

द्विगुणं जाता······आहुत्वअढो······गुण······भागस्य दिवर्धा *X* किं

| १/६६ | ११/२ | फलं···फलं ५

E 1.

(66 recto. 66 verso)

···एकार्घं तु पण्यानां एक-द्वि-त्रि-चतुष्षट्···पण्यानिमानय:

स्थापनं क्रियते

१/१	१/१	द्रं०	१	२/१	द्रं०	१	३/१	द्रं०	१	४/१	द्रं०	१	६/५

······

प्रत्यय त्रैराशिकेन

१/१	द्रं०	१/१	रू०	१२/१	द्रं०	फलं रूप १२
१/१	द्रं०	२/१	रू०	६/१	द्रं०	फलं रूप १२
१/१	द्रं०	३/१	रू०	४/१	द्रं०	फलं रूप १२
१/१	द्रं०	४/१	रू०	३/१	द्रं०	फलं रूप १२
१/१	द्रं०	६/१	रू०	२/१	द्रं०	फलं रूप १२

E 2.

(53 recto, 53 verso)

··· १ १ २	यो०	१ दि० १	६······ १	······

विशेषं तु तत्र गतिं	३ २ २ १	विशेषं	१ २	··· सर्वं गति

योजन | ६ | अनेन गुणये | १८ | अनेन···भविष्यति

प्रत्यय त्रैराशिकेन	१ दि० १	यो० २७ १	दिन ६ आदी योजन ६

	१ १	१ दि० १	···यो० फलं योजन

······ ······	१८ योजन १	८० दिन २० घटिके ३५* घ० दिन	फलं यो० २७ ७
⋮	२७ यो०	२० २० घटिके ३५* घ० दिन	फ० यो ३६ ७

E 3.

(58 recto, 58 verso)

उदा० ॥ षड्-विंशश्च त्रि-पञ्चाशेकोन-त्रिंशैव च ।

द्वा श······षड्विंश चतुश्चत्वारिंश सप्तत्रि ।

चतुष्षष्टि नव···न्शनन्तरम् ।

त्रिराशीत्येकविंश अष्ट···पकम् ।

(२६५३) · २६६२२६४४७०६४६६४···(८३२१८) ·

स्थापनं क्रियते···	१	युवां	१	सूढ	१	दृश्य	२०
	१		१		१		
···	३		१	मण्ड	१	मण्डें	२
		मम्	१		२		
			२				

···त दत्त जातं मण्ड २ | य ५ | सूढे १

E 4.

(21 recto, 21 verso)

······त्रिभाग ·····दिने तथ । त्रि रूप पञ्चभि दिनें ।

एषां द ·····

रू० १ १	रू० १ १	रू० ३ १	दृश्य १०० ।
१ दि० ३	१ दि० २	५ दि० १	

करणम् ॥······कृत्वा:

३	२	३	दृ० १००
१	१	५	१

·········वार्धं तृतीयस्य

जीव लोकातेषां दीनार······कस्य किं भवति ॥

२ दी० १ २	३ दी० १ २	४ दी० १ ।
१ दी० १ २	१ दी० १ ३	१ दी० १ ४

······परिवर्तनं क्रियते | १० / ६ | २१ / ८ | ३६ / १० | दृ० ५०० / १ |

······ प्रक्षे······ ···

E 5.

(22 recto)

······ | ४७३५०० / ६४७ | वर्तित जाता फलं दी ५०० ॥

अस्य प्रत्यय त्रैराशिकेन ।

२ दी० / १ / २	१ दी० / १ / २	१००००० दी० / ६४७	फलं दि ६०००० / ६४७
३ दी० / १ / २	१ दी० / १ / ३	१५७५०० दी० / ६४७	फलं दि ६०००० / ६४७
४ दी० / १ / २	१ दी० / १ / ४	२१६००० दी० / ६४७	फलं दि ६०००० / ६४७

F 1.

[22 verso]

(i)···द्विगुणं द्वितीयस्य प्रथमा···तीय··· । प्रथमा चतुर्गुणं चैव चतुर्थे चैव दत्त्वान् च···शतमेकम् द्वयानुगं ॥ वदस्व प्रथमे दत्तं किं प्रमाणं···स्य ।

| ० / १ | २ / १ | ३ / १ | ४ / १ | दृष्य २०० / १ |

शून्यमेक युतं कृत्वा १ । २ । ३ । ४ । · क्षेप युक्त्या—

फलम् ॥ २० । ४० । ६० । ८० । एवं २ ० ॥ एषां…

| आ० २० / १ | उ०·२० / १ | प० ४ / १ | रूपोणा करणेन फलं २००

(ii) सूत्रम् ॥ यदृच्छ विन्यसे शून्ये तदा वर्गं तु कारयेत् ।

F 2.

(23 recto; 23 verso)

(i)…च त्रिगुणं…

…प्रथमस्य तु किं भवेत् ।

| ० / १ तदा २ / १ | तदा |

यदृच्छा विन्यसे शून्ये…च्छा | १ | तदा वर्गं तु कारयेत् ।

| १ / १ | २ / १ | २ ३ / १ १ | ६ / १… | प्रक्षिपे गुणितं १ | २ | ६ | २४ |

…प्रक्षिप्तं ३३ ॥ दृष्यं विभजेत् | १३२ / ३३ | वर्त्यं जातं | ४ / १ |

…दत्तम् ॥ अतोन्यासः | ४ | ८ | २४ | ९६ | …

एष वर्गं क्रम गणितम् ॥ अथ युति वर्गं क्र…

(ii) सूत्रम् ॥ कामिकं शून्ये विन्यस्तं तदा चैव क्रमे गुणं ।

(i) …कृत्वा चतुर्थ…

…प्रथमस्य तु किं भवेत् ।

०	२ १	३ ३	१२ ४	दृ० ३००
१	१ १	१ १	१ १	१

कामिकं शून्ये पिन्यस्तं कामिकं १ ।। एष न्यस्तम्

तदा चैव क्रमेण गुणितं । १ | २ | ६ | ४८ | एषां यु० | ६० |

अनेन दृष्यं भाजितं | १/६० | ३००/१ | जाता ५ | ए ..

अनेन क्षेपं गुणये | ५ | १० | ४५ | २४० | ..युतं वर्ग गणितम ।।

(ii) उदा० ।। प्रथमस्य न...दत्तं चवै धनम् ।

स च द्वयार्धं युतो दत्तं...

F 3.

[24 recto, 24 verso]

शतञ्चतुश्चत्वारिंश...दत्तं चैव चतुर्गुणम् *X*

किं प्रथमस्य...

०	१	२	२	३	३	४	४	दृ० १४४
१	१	१	१	१	१	१	१	
	२		२		२		२	

शून्येषु १...युतं चैव गुणं ततः युतञ्चैव गुणं कृत्वां कारये गण.. | ५/२ |

गुणम् । उपरे उपरमधे अधं गुणये | १०/२ | सार्धं द्व ...युतं...

स्तीय राश्यागुणणम् । सार्धं स्सप्तभि त्रीणि | ४५/२ | सार्धं त्रय युतं...चतुर्थं

राशि गुणपेम् षड्वं विंशतिभि जाता | २०८/२ | सार्धं चत्वारि यु ...

| २८९/२ | एवं दृष्यं । सर्वं तदेव जातम् —

(ii) ···त्रिसार्धं यु···

···चतुर्गुणं चतुर्थेन नवार्धं युतम् दत्तम् ।

···द्विशता द्वाविंशाधिका किमत्र प्रथमस्य दत्तासित्

| ०/१ ३/२ | २/१ ५/२ | ३/१ ७/२ | ४/१ ९/२ | एकत्वं दत्तम् २२२ |

शून्य···दत्वा । १ । युत गुणित युत क्रमेण जातम् ॥

स्थाप··· | ५/२ | १५/२ | ६७/२ | ३५७/२ | दृश्य २२२ | प्रक्षेपेण

जातम्+२२२ ॥ ··· दृष्याः··· २२२ ॥

(ii) उदा० ॥ प्रथमं न जानामि । दिवार्धं युतं······

F 4.

(25 recto, 25 verso)

०/१ ३/२ | २/१ ५/२+ | ३/१ ७/२+ | ४/१ ९/२+ | दृ० ७८/१ | .

···युतं जातं ५/२ | द्वितीय गुणं १०/२ | ···तृतीय एकत्ने

गुणितम् । युतेन । | १०/२ | २३/२ | ···युतं | ३३/२ | गुणितं

| १३२/२ | ऋणं जातं । पार्य···एश न्यास | ५/२ | ५/२ २३/२ | १२३/२ | दृष्य ७८/१ |

······ | १५६/२ | ···विभक्तव्यं | २/१५६ | ७८/१ | ······

(i) करणम् । शून्य स्थाने···रूपं दत्वा १/१ | युता जाता | ५/२ |

··· १५/२ | प्रथसा तृतीयस्य त्रिगुणं युतं जातं

चतुर्गुणं नवार्धं युतं जातं | २६/२ | एकत्र न्यास··· | ५/२ | १५/२ |

| २२/२ | २६/२ | दृ० ७१/२ | प्रक्षिप्तं | ७१/२ | भक्तं इष्यं जातं

१···अनेन सर्वं गुणितं तदेव | ५/२ | १५/२ | २२/२ | २६/२ | एकत्रम्

एषामपरो विधिः ॥

(ii) उदा० ॥ प्रथम धनं दत्तं न जातं किं तु दिवर्धं युतम् ।

तदाद्वितीयेन द्विगुणं दत्तं पञ्चार्धहीनम् ।

तदा तृतीयेन त्रिगुणं दत्तं सप्तार्ध···

चतुर्गुणं नवार्धं हीनम्

दत्तमेकत्वं त······

| २/१ ५/२ + | ३गु० १ ७/२ + | ४ गु० १ ९/२ + | दृ० २६/२ |

F 5.

(26 recto)

(i) करणम् ॥ शून्य···रूपं दत्वाः युतं जातं | ५ |

··· | $\frac{५}{२}$ | प्रथमा तृतीयं त्रिगुणं प्रथमा

चतुर्थं चतुर्गुणं नवार्धं रहितं । शेषं | $\frac{११}{२}$ | ए···

| $\frac{५}{२}$ | $\frac{५}{२}$ | $\frac{८}{२}$ | $\frac{११}{२}$ | द्वि० $\frac{२६}{२}$ | प्रक्षेप युक्तिः | $\frac{२६}{२}$ |

······भक्तं | $\frac{२}{२६}$ | $\frac{२६}{२}$ | जातं | १ | ·· गुणितं तदेव ।

एवं ऋण राशी भवन्ति ।

(ii) त्रिप्रकारं ··समाप्तम् ॥ शून्यस्थाने रूपं दत्वा । तदनुयुक्तं । गुणित ··

F 6.

(26 verso)

अथ द्यौ ४ । ३० अस्य दलं फ०

अथाष्ट ८ । ३२ दलं० फ०

भू० ३६ ।१६ । २८ दलं० फ०

२४ । १६ दलं० फ०

२४	४	अथ त्रीण्युसारा द···
२८	४	३६।२०। ४ अस्य त्रि···
३२	४	३२।२०। ८ अ···
३६	४	२८।२०। १२ पुन···
भू०३६		२४।२८। १६···

G 1.

(10 verso, 10 recto)

(i) सूत्रं २४

(ii) सूत्रं ॥ कृत्वा रूप क्षयं पार्थ धान्त संगुणनं ततः

प्रवृत्तिर्गुणनं ततः··· विनिर्दिशेत् ॥

(iii) उदा० ॥ त्रिभाग मलदग्धस्य··· त्रिधान्तस्यैव···

अष्टोत्तर-शतानि दत्तं किं शेषं वद पण्डित ॥

१०८	१	१	१
१	१	१	१
	३+	३+	३+

कृत्वा रूप क्षयं पार्थ जाता | ३२ | शेष ॥ प्रथम ध्धान्ते क्षयं | ३६ | शेषं | ७२ | द्वितीय ध्धान्ते क्षयं | २४ | शेषं ४८ तृतीय-ध्धान्ते क्षयं | १६ | शेषं | ३२ |

प्रत्ययं क्रियते । स्थापनम्

०	१	१	१	भा० शेषं	३२
१	१	१	१		१
	३+	३+	३		

फलं मूला १०८ ॥ अथ-

सजाति क्रिया

(i) ··· ० ··· त्रिभि त्रयष्ट-भाग संयुतं······

१
१ तदाष्टोत्तर-क्षता किं | २७ / ८ | १ / १   १०८ / १ | फ० शे० ३२
३+
१
३+ ··· यद्येकस्य त्र यस्त्रयाष्ट भाग तदा द्वा-
१ ।

३ त्रिशानां किमिति | १ १ | ३ ३ ८ | ३२ फलं १०८ १ |

(ii) उदा० ॥ सकृद् धान्तस्य लोहस्य दशांशा क्षीयते-स्वयम् ।

सप्तते द्विगुणा च किं शेषं वद पण्डितः" | ३ १० | १४० |

कृत्वा रूपक्षयं पास्थमिति·· रूपं | १ | ३ १० | क्षयं कृत्वा

जातं शेष | ७ १० | मूलं | १४० १ | अनेन गुणितं जातं । ९८ | क्षयं | ४२ |

एवं | १४० |

······ | ७ १० | १ १ | ९८ १ | फलं १४० |

G 2.

(11 recto, 11 verso)

(i) प्रत्ययः | ० १ १ ३ + १० | ···

(ii) उदा० ॥···पला क्रीते पल त्रिभागं क्षय व्रजति ।

अष्टादश······थतां ब्रूहि ॥

| १ ३ | ३ २ भा | १८ १ |

करणम् ।अद्ध्यर्धं पलंश्छेदेभ्य इदं | २ ६ | कृत्वा रूप

क्षयं रूपं १ क्षयं कृत्वा जातं | ७/६ | १८/१ | गुणितं जात १४ |

क्षयं | ४ |

प्रत्यय त्रैराशिकेन ॥

अद्ध्यर्धं पल क्रीते त्रिभाग क्षय गच्छति ।

अष्टादश पल क्रीता किं क्षयं वद पण्डित ॥

१ १ २	१ ३	१८ १	फल ४ १

पुन त्रिभाग दिवर्धं तदा चतुभि किमिति

१ ३	१ १ २	४ १	फलं १८ १

(iii) उदा० ॥ चतुर्भाग मल दग्ध सुवर्ण शतपञ्चकम् ।

....अथ प्रत्यय......

०	१५८ सू०
१	१ तो ५*
१/४ +	१/६४

फल मूल ५०० ॥ पुनरेव प्रस्तार क्रमम्

१/४ +
१ +
१/४ +

५००	१	१	१	१	फलं शेष
	१	१	१	१	
	४ +	४ +	४ +	४ +	तदिति

शेष १५८ तो० १ शे १/६४

अन्यं चतुर्थ प्रत्ययं क्रियन्ते

०	१	१	१	१	भा० शेष १५८ १ ५* १ ६४	फलं ५०० ॥
१	१	१	१	१		
	४ +	४ +	४ +	४		

आद्यं क्षयं १२५ शेषं ३७५ । द्वितीये क्षयं ९३ ती० ३ माश ९

शेषं	२८१ / १ / ४	क्षयं	७० / ५ / १६	शेषं	२१० / १५ / १६	क्षयं	५२ / ४७ / ६४

शेषं | १५८ / १३ / ६४ | एष सर्वत्र कर्त्तव्या ॥

G 3.

(12 recto, 12 verso)

···प्रस्थ··· ···मधुनास्तथाः

अम्भस··· ··· ···

क्रीत्वा रूप क्षयं पास्तमितिः तत्र क्षयं : १ास्तंः इति : तत्र क्षयः

रूपं गुण्य शेषं | ३ / ४ | ३ / ४ | ३ / ४ | ३ / ४ | ४ / १ | गव्यूति गव्यूति गत्वात्प्रस्थं

पिवेत् ·· गव्यूति योजनम् । चतु प्रस्थै आढकम् । तदा धान्तसोर्गु··· ···

ततः | ८१ / २५६ | आवृत्ति प्रवृत्तिर्गुणनं ततः | ४ | अनेन गुणितं जातं | ८१ / ६४ |

एष मठव भागाभागे हृते लब्धम् । मधु प्रस्थ १ कु० १ शे | १ / १६. | अम्भ

भागा प्रस्थ २ कुडव २ शे | १५ / १६ | एवं | ४ | ···कुडवोक्ति प्रक्षेपके आढका

षोडश कुडवा भवन्ति | १६ | अतो म···शेषम् १२

(i) ··· ······प्रस्थ कुडवा | ४ | ३ | शेष चत्वार···

कुडवः | २ / १ / ४ | २ / १ / ४ | शेषा च कुडवा पीता । म० | ७ / १ / ४ + | ९ / १ / ४ | पुन

चत्वारि कुडवा भक्तं शेषं | $\frac{८१}{१६}$ | $\frac{१७५}{१६}$ | जल भागम् । मधुकुडव

| $\frac{५}{१}$ शे० $\frac{१}{१६}$ | जल कुडव | $\frac{१०}{१}$ शे० $\frac{१५}{१६}$ | एवं कुडव १६ ॥

(ii) उदा० ॥ दत्वा शुल्कं चतुर्भागमष्टौ आनीत कुङ्कुमा ।

चतु शुल्क शालैस्तु किं शेषं वद.पण्डित ।।

| ८ / १ | १ / १ / ४ + |

करणम् । कृत्वा रूप.क्षयं पास्तं पास्तं | $\frac{८}{१}$ $\frac{३}{४}$ | गुणितं

जातं | ६ | शुल्के | २ | शेषं | ६ / १ | १ / १ / ४ + | अनेन गुणितं जातं

| ४ / १ / २ | क्षयं | १ / १ / २ | शेषेण | ४ / १ / २ | १ / १ / ४ + | दत्वा गुणित जाता | $\frac{२७}{८}$ |

••• •••

(i) उदा० ॥ अज्ञातारम्भ-लोहस्य त्रि-चतुः-पञ्चका क्षये ।

सप्तविंशति पिण्डस्य त्रिधान्तशेष्य दृष्यते ।

किं सर्वं वद तत्वज्ञ क्षयं च मम कथ्यताम् ।।

| $\frac{१}{३}$ | $\frac{१}{४}$ | $\frac{१}{५}$ | शे० $\frac{२७}{१}$ |

करणम् । कृत्वा रूप क्षयं पास्थ | $\frac{२}{३}$ | $\frac{३}{४}$ | $\frac{४}{५}$ | गुणितं

जातं | $\frac{२}{५}$ | रूपक्षयं | $\frac{३}{५}$ | अनेन शेषं भक्तं शेषं | २७ |

भक्तं जातं ४५ अस्य सप्तविंश । पात्य शेषं १८ । एत क्षयम् ॥

(iii) उदा०. परिक्षीणस्य लोहस्य त्रिधान्तं पञ्चमाषकम् ।

न ज्ञायतेत् प्रवृत्तकां न तु शेष प्रदृष्यते ।

प्रवृत्ति शेषं यो पिण्डं केवलं विंशति स्थितम् ।

अज्ञातकां प्रवृत्तीस्यां किं वा शेषं वदस्व मे ॥

| १/३ | १/४ | १/५ | कृत्वा

G 4.

(13 recto, 13 verso.)

(i) | ८/१   १/१ + १/४ + १/४ + १/४ + १/४ + | गुणितं जातं | ८१/३२ | पुनान्यं

| ८   ३/४   ३/४   ३/४   ३/४ : फलं | ८१/३२ | पुनान्यं | ८/१   १/१ + १/४ + १/४ + १/४ + १/४ + | फलं

| ८१/३२ | पुन प्रत्ययं | ०/१   १/१ + १/४ + १/४ + १/४ + १/४ + | भा शे. ८१/३२ : फलं कुङ्कुम ८॥

(ii) उदा० ॥ त्रिभाग षड्भाग पञ्चांशं गुडपिण्डाष्टभारकम् ।

किं शेषं दत्तभिर्भवेत् ॥ ...

८	२	५	गुणितं जातं	३२	एतत् फलम् ॥
१	३	६		९	

(iii) उदा० ॥ चतु ॐ पञ्चक लाभेन दश द्रोणात् प्रयोजित ।

तद्वै त्रिभिस्तु किं लाभं कथ्यतः गणकोत्तम ॥

१०	५	५	५	गुणितं जातं	१२५०
१	४	४	४		६४

(i)	०	भा० शे० १९	फलं १०॥	०	फलं द्रों० १९ आ
	१	१		१	
	१	आ० २		१	२ प्र० ० कु० २
	४	४* द्रां०		४	
	१	प्र० ०		१	
	४	४* आ० प्र०		४	
	१	कु० २		१	
	४	कु० ४* प्रस्थ		४	

(ii) उदा० ॥ कस्याप्यज्जंकस्य षष्ठि स्वदलेन क्षयं गत ।

पुन वृद्ध्या त्रिभागेन स्वपादेन ततोज्झितम् ॥

वृद्ध्या तु पञ्चभागेनस्तथा वृद्धि द्वयो गतम् ।

का वृद्धि ...स्या किं वा शेषं तदुच्यताम् ॥

६०	१	१	१	१	रूप ला...जाता ३६ ॥
१	१	१	१	१	
	२ +	३	४ +	५	

प्रत्ययं पुनस्यैव	०	१	१ १	१	भा०	३६	फलं ६० ॥
	१	१ +	१ १ +	१		१	
		२	३ ४	५			

पुनान्य प्रत्ययं | ६० १ १/२ + १ ३ १ ४ + १ ५ | फलं ३६ ॥ ...मूलं न ज्ञायते

फलम्

+ +
६० १ १ २ १ ३ १ ४ १ ५

G 5.

[14 recto, 14 verso]

.. ..

यस्य तन्मयता चक्षु

| १/३ | १/४ | १/५ | अपहृत शुल्क पिण्डं २४ ॥

करणम् ॥ कृत्वा रूपा क्षयं पास्त | २/३ | ३/४ | ४/५ | जातु संगुण्य

जातं | २/५ | एतावदपि रूप संशुद्धा जातं | ३/५ | अनेन भक्त्वा शुल्क

पिण्डं गुणितं जातं | ४० | एष पिण्डम्

प्रत्ययं | २/५ ४०/१ | गुणित जातं १६ शेषं | २४ | एवं | ४० |

अन्त्यमस्य प्रत्ययं ४० | १ १/३ + १/४ + १/५ + | फलं १६ क्षयं २४ एवं ४० ॥

(i) उदा० ।। गुडपिण्ड ज्ञात तुल्योश्चतु··· अव्ये गुडम् ।

त्रिचतु φ पञ्च षड् वृद्ध्या चत्वारिंश (भ*) वे क्षय

G 6.

[15 verso, 15 recto]

···प्रवृत्ति भवेत् सखे"

$\left| \frac{१}{३} \right| \frac{१}{३} \left| \frac{१}{३} \right| \frac{१}{३} \Big|$ शे $\frac{१६}{१}$

करणम् ।। धान्तशो घातितं तेन । रूप क्षयं कृत्वा जातं

$\left| \frac{२}{३} \; \frac{२}{३} \; \frac{२}{३} \; \frac{२}{३} \right/$ गुणितं $\left| \frac{१६}{८१} \right|$ भक्तं $\left| \frac{८१}{१६} \right|$ शेषेण गुणये ।

शेषं $\left| \frac{१६}{१} \right|$ गुणित जाता $\left| \frac{८१}{१} \right|$ ···प्रवृत्तिरित्यर्थः ।। अथान्य

विधि कला संवर्णे

चतुर्धान्ति लोहस्यैकाशीतिश्च दत्तवान् ।

किं शेषं वद धर्मज्ञ य गणिते कृतं श्रमम् ।।

| ८१   १   १   १   १
  १   १   १   १   १ +
     ३ + ३ + ३ + ३ | फलं शे० १६ ।।

पुन प्रत्ययं क्रियते मूलं न ज्ञायते

| ०   १   १   १   १   भा० शे० १६
१   १ + १ + १ + १ +   १
    ३   ३   ३   ३ | फलं लोहपल ८१ ।।

उदा०...

कश्चि यदि शक्य तदुच्यताम् ॥

एतन्मे संशयं प्राज्ञद् धान्तक्षयं विचारणा:

| २ | ३ ४ | क्ष० शे० ३२ |
| ३ | ४ ५ | १ |

करणम् ॥ धान्त संगुण्य गुणितं जातं | ३/५ | रूपं दद्या | ८/५ |

भागे हृते लब्धं भक्... | ५ ३२/८ १ | फलं २० एष सा प्रवृत्ति ॥

शेषं १२... ३२ ॥ पञ्चविंशतिम सूत्रम् ॥ २५

G 7.

[16 recto]

(i) विभक्तं जातं | २ शे० १०/९ १ | ६/७ |

अनेन गुणितं जातं | ६/७ | भागे हृते लब्धम् १२ ॥

अस्य प्रत्यय त्रैराशिकेन

७	१	१०	फ० १२
६	१	१	६
	२		७

(ii) उदा० ॥ माक्षिक ग्-घटकस्यैव द्वि-त्रि-भाग प्रवर्धितम्

द्वितीये द्वि-पञ्चमो-भागो तृतीये द्वि-सप्तकोद्भवम्

चतुर्थे द्वि-नवं भागमेवं जात पल त्रयम् ।

बभूवासौल्किर्कै हृत्वा किं सर्वं वद पण्डित ॥

२	२	२	२	शे०	३
३	५	७	९		१

धान्तसो···इति । कृत्वा

H 1.

(16 verso)

······सूत्रम्······

(i) इदानि सुवर्णक्षयं वक्ष्यामि···स्येदम्

(ii) सूत्रम् ।। क्षयं ❀ संगुण्य कनकास्तद्युतिर्भाजयेत् ततः

संयुतैरेव कनकैरेकैकस्य क्षयो हि सः ।

(iii) उदा० ।। एक-द्वि-त्रि-चतुस्संख्या सुवर्णा मापकै ऋणै ।

एक-द्वि-त्रि-चतुस्संख्या रहिता समभागताम् ।।

स्थापनं क्रियते । एषां

१+	२+	३+	४+
१	२	३	४

करणम् -"क्षयं संगुण्य कनकादिभि' क्षयेन संगुण्य जातं । १ । ४ ९ १६ ।···। एष युति । ३० । कनका युति १० अनेन भक्त्वा लब्धम्······

H 2.

(17 recto., 17 verso)

(i)

१	१			
१०	३०	४	फ० मा० शे०	१२
१	१	१		१

(ii) उदा० ।। एक-द्वि-त्रि-चतुः संख्या सुवर्णं प्रोज्झिता इमे

माषका द्वि त्रिताञ्चैव चतुः पञ्चकरांशकम् किं क्षयम्

१	२	३	४
१	१	१	१
२	३	४	५

करणम् ॥ "क्षयं संगुण्य कनका" एष स्थापयते ।

१	२	३	४
२	३	४	५

"तद्युतिर्भाजयेत् ततः" हर साम्ये कृते युतम् | १६३ / ६० | "संयुते X कनपी"

भक्त्वा तदा कनक | १० | अनेन भक्तं जातं | १६३ / ६०० | एष एकैक सुवर्णस्य क्षयम् ॥

प्रत्यय त्रैराशिकेन…

१०	१६३	१	फ०	१६३
१	६०	१		६००
१०	१६३	२	फ०	१६३
१	६०	१		३००
१०	१६३	३	फ०	१६३
१	६०	१		२००
१०	१६३	४	फ०	१६३
१	६०	१		१५०

क्रमेण द्वय माषादि उत्तरे एक हीनताम् ।

सुवर्णं ये तु सम्मिश्र्य कथ्यतां गणकोत्तम ॥

स्थापनम्

४ +	५ +	६ +	७ +	८ +	९ +	१ +	२ +	३ +
५	६	७	८	९	१	२	३	४

"क्षयं संगुण्य" जातं । २० । ३० । ४२ । ५६ । ७२ । ९० । २ । ६ । १२ ।

एषां युति । ३३० ॥ कनकानां युति ४५ । अनेन भक्त्वा लब्धं | ३३० / ४५ |

पञ्चदशभागेच्छेद क्रियते । फलं | ७ शे० १ / १ ३ | एष एकैक माषक क्षयम् ।

प्रत्यय त्रैराशिकेन | ४५ / १ | ३३० / १ | १ / १ | फलं २२ / ३ |

एवं सर्वेषां प्रत्ययम्

H 3.

(18 recto, 18 verso)

(i) (सूत्रम्) ।। अप्राप्त संगुणा कति·· कञ्चनानि ततोज्झितम् ।

... ...

कञ्चनै यद् भवेल्लब्ध स क्षय जात माषक ।।

(ii) उदा० ।। एक-द्वि माषको प्राप्तो द्वौ च प्राप्तं च पञ्चभि ।

त्रयश्च कतिभि ф प्राप्त ·डेव नि केवलम् ।

चतुर्भिमषिकैर्हीनं कति दृष्ट्वा मया सखे ।

त्रयश्चकतिभि ф प्राप्ता सुवर्णा माषको वद: ।

१	२	३	६
२	५	०	४+

करणम् ।। "अप्राप्त संगुगा कतीदि" ति | ६ / १ | अप्राप्त कति चत्वार । ४ । संगुण्य जातं । २४ । "कञ्चनानि ततोज्झितम्" द्वाभ्यमिक पञ्चभि द्वयं संगुण्य जातं २ | १० / १ तद्युति १२ । हित्वा २··· हित्वा जातं शेष १२।। अप्राप्त गण्डिकै······

······ ·· अष्टविंशतिम सूत्रम्

(i) सूत्रम् ।। अनैस्सगुण्य कनका तत् पिण्डञ्च विशोधयेत्

सु ❀ वर्ण कनकाभ्यास्ता राशि शेष विभाजयेत्

अप्राप्त गण्डिका शेष शुद्धेन कनकेन तु ।

यल्लब्धं तत् प्रमाणं तु गण्डिका या विनिर्दिशेत् ।।

(ii) उदा० ॥ एक-द्वि-त्रि-चतुस्संख्या अप्राप्त माषकानि तु

एक-द्वि-त्रि-चतुस्संख्या एकत्रावर्तिता किलः ।

गण्डिका ज्ञात कनका अनेकादश माषकं ।

अप्राप्त ज्ञात कनकं प्र··········यः ।

१ १	२ २	३	४	०

करणम्·········।

J 1.

(30 recto; 30 verso)

··· ··· ··· ···

सूत्रम् । एक युत नर······सर्वष् षड्भि प·········

······अनेन लब्धं ······हीताप्रथम्

३६	४२	४८ ७८	५४ ७	६

······सदृश क्रि······भाग हार क्रियने | २३४
७० | ···

···तुलाढे | ३
२४
७० | मुद् गाढे १
४७ | ······क्रियते

J 2.

[65 recto,]

एतत् = काल··· भिर्मनुष्या य···लग्यभ्सिः··· ❀

अपर प्रश्नः

यद्येक पुरुषस्य द्रम्माष्–षट् त्रिंशभिर्विनै जीवलोका । तत्कार्यं प्रस्तुता ··

स्सप्तीनां···नांपाकराक्षकानां द्रम्मैष्–षड्भि कांत दिना जीव लोकं भवति···

करणम् । आदौ तावयद्येक पुरुषस्य द्रम्माष्–षट्–त्रिं शभि···जीव्य·:

तत् सप्ततीनां किम्

| १ पु० द्रं ० ६ | ३० दि० | ७० पु० | फलम्··· |
| १ १ | | १ | |

द्रम्मा···त्रीणि शतर्धासा······

J 3.

(41 recto; 41 verso)

···द्रम्मा अष्ट द्वा–चत्वालिंशभिर्दिनै । तत् सप्तति···

| य ४२ दिने | द्रं० ८ जीव्या | ७० पुरुषा ४२··· |
| १ | १ | १ १ |

द्रम्मा ५६० ॥ यदि पञ्च-शत-षष्ट्याधिक···द्व-चत्वालिंशभि तद् द्रम्मै, अष्टभि कति दिनां···

·· २ अधेदापये दत्त: | १७
८
२
३ | अधेनो परि सं···उपरिम

राशि द्वय गुणये | ५१
६
२
३ | ······ उपरि युक्त क्रियते एक पञ्चाशानां ५१
६

स्थापनम् १ | ५३ | फलं आ १७ त्रि···२

K

(51 recto.)

(i) उदा० ।। को राशि पञ्च युता मूलदः सा राशिस्सप्त हीन मूलद को सो राशिरिति प्रश्नः

०	५	यु० मू०	०	सा ०	७+	मू० ०
१	१		१	१	१	१

करणम् । "युतहीनं च—मे कत्वं" | १२ | तद् दलं | ६ | द्वि हीणम् | ४ | दलं | २ वर्गं | ४ | "हीने युतिं च कर्तव्या" । हीनं | ७+ | अनेन युति | ११ | एष सा राशि ।। अस्य प्रत्यानयने य क्रियते

११	यु० ५	मू० ४	११	७+	मू० २
१	१	१	१	१	१

पञ्चाशं सूत्रम् ५०

सूत्रम् । गवां विशेष कर्त्तव्यं धनं चैव पुनः…… …

.. ..

L 1.

(60 recto, 60 verso)

(i) ··एकोनविंशतिम् | गावी १०/१ | रूप ८/१ | ·· | विवरितास्ति ।।

एकपञ्चाशम सूत्रम् ५१ ।।

(ii) सूत्रम् ।। आय व्यय विशेषं तु विभज्य दृष्य संगु ❀ णम् ।

यल्लब्धं सा भवेत् कालमयं प्रश्ने···य विधि ।।

(ii) उदा० ॥ द्वि-दिने आर्जये पञ्च त्रि-दिने नव भक्षये

भाण्डागारं तस्य त्रिंश किं कालं आर्जं भक्षणम् ।

दी०	५	दीनार	९		६०
दि०	२	दिन	३		३०

करणम् । "आय-व्यय विशेषं तु" । तत्रायं | ५ २ |

(i) ···बोदि । फलं १८० | द्वापञ्चाशम सूत्रम् ५२ ॥

(ii) सूत्रम् । अह द्रव्य ❀ हराशीत तद् विशेषं विभाजयेत्

यल्लब्धं द्विगुणं कालं० दत्ता सम-धना प्रति ॥

(iii) उदा० । त्रि-दिने आर्जये पञ्च भृतको-मेक पण्डितः

द्वितीयं पञ्च दिवसे रस मार्जयते बुधः

प्रथमेन द्वितीयस्य सप्त दत्ता निधानतः

दत्वा सम-धना जाता केन कालेन कथ्यताम्

५	रू	६···
३	दि	५···

L 2.

(61 recto, 61 verso)

(i) ·· अनेन कालेन समधना भवन्ति ॥

प्रत्ययं त्रै-राशिकेन क्रियते

३	५	३०	फ० ५०	प्रथमे द्वितीयस्य सप्त दत्ता ।	७
१	१	१			
५	६	३०	३६	शेषं ४३ ।······	४३ ।
१	१				

४३ । एते समधना ज्ञाता ॥

(ii) उदा० ।। राजपुत्रो द्वयो केऽच नृपतिस्सेव्य सन्ति वै:

मेकास्याह्नि द्वयष् षड् भागा द्वितीयस्य दिवर्धकम् ।

प्रथमेन द्वितीयस्य दश दीनार दत्तवान्

केन कालेन समताम् गणयित्वा वदाशू मे ।।

१३	३	दत्तं १०
६	२	१

करणम् ।। "अह् द्रव्य विशेषञ्त्र" । तत्व

(i)

१	१३	३०	फ ६५	प्रथमेन द्वितीयस्य
१	६	१		
१	३	३०	फ ४५	
१	२	५		···दत्त जाता ।

५५ | ५५ ।। समधना जाता ।।

(ii) सूत्रं त्रि-पञ्चाशम: सूत्रम् ५३ ।। ❀

सूत्रम् ।। विक्रयेन क्रयं भाज्यं रूप हीनं पुनर्भजेत

लाभेन गुणये तत्र नीवी भवति तत्र च ।।

(iii) उदा० ।। द्विभि *X* क्रीणाति यस्सप्त विक्रीणाति त्रिभिष् षट्

अष्टादश भवेद् लाभा का नीवी तत्र कथ्यताम् ।।

७	६	१८	लाभा
२	३	१	

करणम् । वि······

L 3.

(62 recto, 62 verso)

नीवी जाता । स्य प्रत्यय त्रैराशिकेन ।।

यदि द्विभिस्सप्त लभ्यते । तदा चतुर्विंशतिभि X किम् ।

| २ / १ | ७ / १ | २४ / १ | फलं रू० ८४ ॥

(i) अस्य विक्रयं क्रियते । यदि षड्भि त्रय लभ्यते तदा चतुराशीतिभि X किम् ।

| ६ / १ | ३ / १ | ८४ / १ | फलं ४२ । मूलं ४ । पात्यशेषं १८ एष लाभा:

चौपञ्चाशम सूत्रम् ५४ ❀

(ii) सूत्रम् ॥ विक्रयं भाज्ये चैव गुणयेत् क्रय पिण्डताम् ।

रूपो ने मूल गुणये लब्ध लाभं च प्राप्यते ॥

(iii) उदा० ॥ द्विभि क्रीणाति यस्सप्त विक्रीणाति त्रिभिष् षट्…

(i) ‥ | २ / १ | ७ / १ | २४ / १ | फ० ८४ / १ | अथ विक्रयं | ६ / १ | ३ / १ | ८४ / १ |

| फ० ४२ / -१ | … २४ । पात्य शेषं १८ । एष लाभम् ॥

पञ्च पञ्चाशम सूत्रम् ५५

(ii) सूत्रम् ॥ विक्रयं भाजये चैव गुणयेत् क्रय पिण्डवत्

विभक्तं स च कर्त्तव्यं गुणये मिश्रकं बुध:

यल्लब्धं सा भवेन्मूलं यच्छेषं लाभपिण्डताम् ।

(iii) उदा० ॥ त्रिभिश्च लभतेरष्टौ चतुर्भिश्च विक्रयंष् षट

स मूल लाभमुत्पन्ना शतं षष्टि त्रिमिश्रितम्

किं मूलं कश्च लाभं च कथयेद् गणकोत्तम: ॥

| ८ / ३ | ६ / ४ | मिश्र १६३ / १ |

करणम् । "विक्रयं भाजये चैव गुणयेत्"

… … … … …

L 4.

(63 recto, 63 verso)

(i) ··· | ८/१ | ३/१ | २४०/१ | फल ६०/१ | ··

| ६/१ | ४/१ | २४०/१ | फलं १६०/१ | मूलं | ६० | पात्य शेषं | ७० |

(ii) षट् पञ्चाशम सूत्रम् ५६

॥ विक्रयंञ्च विभक्तव्यं गुणितं क्रय राशिवत्

कृत्वा रूप क्षयञ्चैव विभक्तं मूलं आप्नुयात् ॥ ❀

(iii) उदा० । पञ्चभिश्चतु वर्गं तु गृहीतं केन मानव

केन ष् षट् विक्रीतं ष् षट् पञ्चाश ऋणं कृतम् ।।

क्रय विक्रय संगुण्य नीविस्तस्यैव कथ्यताम् ॥

| १६/५ | ६/१ | ऋणम् ५६+/१ |

···भाजये चैव | १/६ |

(i) पुनास्य विक्रय | ६/१ | १/१ | ३८४/१ | फलं | | ६४ | मूलं | १२० |

चतुष्षष्टि पात्य शेषम् | ५६ | एष ऋणं क्रि···

सप्तपञ्चाशम सूत्रम् ५७ ❀

(ii) सूत्रम् । वस्त्र शुल्कं यद् भवति तद्···हृत वस्त्रताम् ।

त्रैराशिक विधानेन शुल्क विक्रय तत्वतः ॥

(iii) उदा० ॥ पटस्य शुल्क विंशांशं क··· त्रिश्-शतम् ।

पटकानां पण कृते द्वौ पटौ हृत शौल्किकौ ।

···मूल्यं पण दशस् तेषा: किं मूल्यं ··

M 1.

(20 recto, 20 verso)

(i) १ २० रत्ति	धा० १	सु० १ छे० ८०* रत्ति-सु० रत्ति १	फ०घ० ४
१ १	१	१ १ १	
४	अ० ०		
	य० ४	य० १ पा० ३ मू० १	
	१		
	४		

पुन तृतीयस्यैव २ २०	१	१
१ १	३	
४	१ १ छे० १/२*	
	२ २	
	१ १ छे० १*	
	४ ४ ४	

···छेदं ६ धा० —द्र० फ० धा० ४ य० १·· पा० २ मू० १ ॥

सुवर्णस्य मानं समा···

(i) उदा० ॥ स पञ्चनव भागानि दिनानि त्रयोदश: ।

··· नाम् किम् ॥

(i) मू० १२०००···

उदाहरणम् । सर्पोष्टा-दश हस्तो प्राविशत्याधांगुलम्

स नव भाग ·· ति एक विंशति भागमपहरन्ति ।

प्रतिदिनेन: किं कालेन विलं संप्राप्यते ॥

१ १	१ १	१८ छे० २४ + अं०हृ०	फलं व०२ मा०४ दि०
२ २१ +	१ ३६०	१ १	१०३
१			
६			

उदाहरणम् । कीट *X* : किलार्धंगुलं दिवसे दिवसे···

··· ··· ···

M 2.

(33 verso, 33 recto)

उदा० ॥ सुमेरु पृथिवी शंकु सुराणां परिमाश्रयम् ॥

आरु······*X* कश्चितरसासुरमदिरम् ॥

सततं सप्तसार्धाणां सपागध्य··· ॥

सत्रिभाग त्रि-पञ्चांश नित्यमेवञ्च गच्छति ।

योजनानां शतगुण सहस्राणि चतुराशीतिरुच्छ्तम् ।

केन कालेन सौ गच्छे वदमेत सुनिश्चतम् ॥

···७	दि० १	यो० ८४०००	अधच्छेदं ३६० दि०
१	१	१	
···२			

(ii) उदा० ॥ दीनार कोनामविशा त्रि दु*X*खार्जनीयं सुखभोजने च ।

तस्यार्धमर्धञ्च यदर्धंमर्धं तने देव गुरु प्रसादं कृपणधन भुक्तम् ॥

१	१ १ १ १ १ १	१०८	फ० दी० १ धा ८ व०
१	२ २ २ २ २ २	१	

(ii) उदा० ॥ अर्धं ''स्तारं नव रोमशतानि च ।

द्वादशस्त्रीति--चर्माणि कति रोमा···

		१२ २४ / १ १		
६००	१२ २४		१२ २४	फ० रोम
	१ १		१ १	

M 3.

[32 recto, 32 verso,

(i) चन्द्रा निभाण

······तु गगनं नीत रावणे । ग······यं त्यक्त सुतयशेतया । साकैकेन परावर्तं धनुर्भाग शप···वणे पतमाणसो दश भागं निधार्यंते । एवं तत परिमाण हीय मानं तु नित्यशः कियतस्तुपरावर्तं भूमिं प्राप्यते ज···

ध० १ / १ / ५	१ / १०+	परा० ८	योज० ३० / १	च्छे० ८००० / १	यो० ज०

फलं परा० २१८१८१ शे | ६ / ११ |

(ii) उदा० ॥ नागश्वच्छन्द गामि द्रत च दश

(i)

१०० / ६ / ८* / ३ / ४००० / ५ / २४*	ऊर्धच्छे० ७६८००० अ०—यो० / १ व० ४२६८६७ मा० १ दि० ४	१ / अधच्छेदं ७६८००० फलम्

(ii) उदा० ॥ व्रज···चरीश्वाक्ता पतितं भूमि तले पटम् ।

त्रिशतांश्य···नान्तु सप्त योजन हीयते ।

चतुर्दशस्तु कोटि हूयत पञ्च · षष्टि च ।

कै दिनै भूतले प्राप्य वद मे गणकोत्तम ॥

न्यास स्थापनं क्रियते ।

M 4.

(36 recto, 36 verso)

(i) ह्या पञ्च त्रिगुणित सखे

...... एष देश प्रमाणं समाप्तम् ।।

(ii) उदा० ।। स ' लवणास्य राशे कोष्ठतां वा कृतां हरे ।

एषां चैकां राशि पुनसु ... प्तधा नीता ।

सप्तानां मपि चैका राशिस्तुलितानि ।

पञ्च सप्तत्या '' सहस्र' भवेत सप्ताष्ट गुणं किम्

रा १ / १	१०७५ / १	५६ / १	अधच्छेदं २०००* प० भा० । फ० भा १०

प० २००

एष राशि लवण प्रमाणम्

(iii) काकिनी दश भागस्य दद्यादष्टा दशीति...।

तस्यां विंशति भागस्व शत भागं प्रयच्छति ।

नरो वक्षश...

(i)	य० २ / १	१ १+ / १ ३६०	यो० ५ छे० ४६०८००० + य० यो० / १	फ० य०

२१३३३ मा० ४

(ii) योजनस्य त्रिभागार्धं स त्रिभागपदोनकम् ।

या नौ दिनत्रिभागेन ' गेन गच्छति ।

सा पुनः पञ्च भागार्धं योजनस्य तथाष्टमम्

...ति निवर्त्तन्ते वायुवेगवलाहृता ।

योजना नांष्टौ तर शतं केन कालेन गच्छति ॥

दि १	मा १	१	गु	१	१		१	३ भा		
३	३	२		५	२	+	८			

M 5.

(34 recto, 34 verso)

[i] ख गा एकाछा भुक्त प्रसृतिं चैवमेव च ॥

·· ष्टौ वद सखे किं खगं वद सुन्दरि ॥

प्र० १	ख० ११	खा ५७६०	फलं खग* ६३३६०
१	१	१	

एष बाहु प्रमाणम् ॥

(ii) कश्चित् पुमाँ सुवर्णस्तु कला पाद युतं यवम् ॥

प्रत्यहं सूलिने शुद्धि किल दत्तवान् ।

पञ्चाब्दै मासमेवन्तु दिनै पञ्चदशस्तथा:

दत्त्वा···स्य सर्वाग्र ज्ञातुमिच्छामि तत्त्वत ॥

दि १	१ ६* भा	५	च्छेदं १६२* यव-तोल
१	१ १		
	१ ४		
	४*		

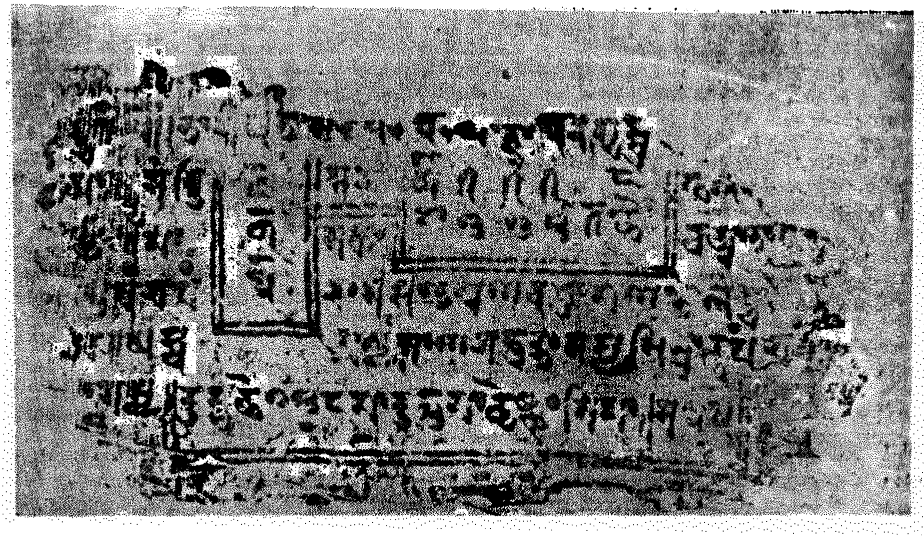
(i) ··· ···

चिन्निताङ्गः । तानीयतासर-परम्परया।र्जुनेन गृध्र···तया

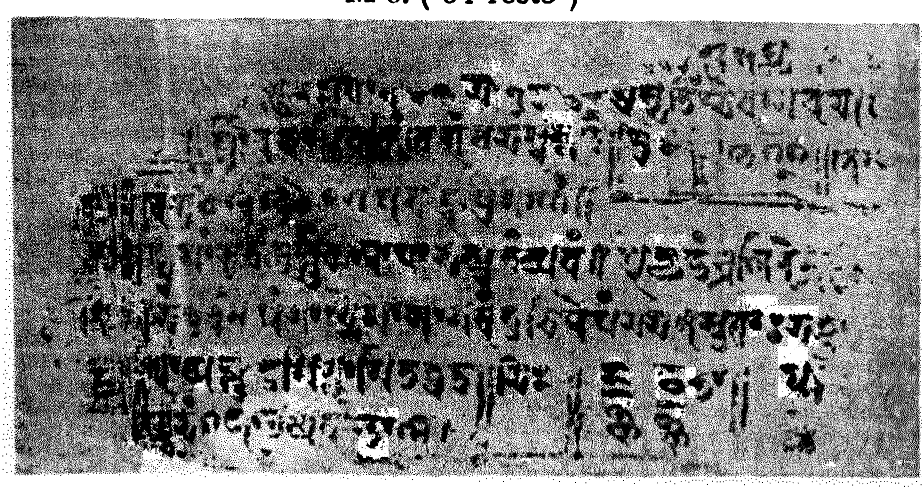
स्पृशन्ति	१	श० १	योज० ७७७ १ ८	फलं—६४०
	८	१	२२२ ७ च्छे	
	१			
	६			
	१			
	५			

Specimen of the Bakhshali Script

M 5. (34 verso)



M 5. (34 recto)



(i) माशकार्धं युतो छयन्त --विस्त पञ्चपञ्चाश सतेरेण वष्ष गर्णं

लब्धं---त्र कथयस्व मूल्यं शाण चतुर्भागस्य सिद्धार्थं पञ्चभागस्य

कु० १ छे० १२८* मा० कु० १ मा० छे० ४०* सि०-मा०	स० ५५
१ १ २	१
२	

M 6.

(37 recto, 37 verso)

(i) ···सूर्यमानस्य

दिवाकरस्य घटिकै X किं प्रयातस्य वद निश्चितम्

३० मु० छे २* घ०-मु० ५००,०००,००० घ० १	फ०यो० ८३३३३३३३
१ १ १ १	

(i) भानो रथं सुर महोरग सिद्धसंघै विद्याधरै ϕ परिवृतम् ··

अहोरात्रौ । कोटी शतार्धं स रथं प्रयास्यात् तद् ब्रूहि शास्त्र-

कुशलो···वर्त्तम् ॥ मुहूर्त्तमे केन किं गच्छे ब्रूहि मे गणकोत्तमा ॥

| ५०००००००० घ० २ | | फ०यो० १६६,६६६,६६६⅔ |

(i) ·· भागे भवेद् राशि ।

ऊर्ध्वच्छेदं १०८००० विलिप्तानां···लिप्ता ५

(ii) पञ्चार्धं संवत्सरे भुक्ते ग्रंशैका यदि भानुजः ब्रूहि···क तत्वज्ञ

समश्व वासरेण किम्

२	रा० १	१ अं० १
१	१	१ ३६०
२		

ऊर्ध्वच्छेदं १०८००० विलिप्तानां राशि । अधच्छेदं १ विलिप्ता लिप्त ॥
६०

फलं विलिप्ता २ । एष ग्रह गतिम् ॥

(ii) उदा० ॥ राज युधिष्ठिरो नामः पाण्डुवंश

M 7.

(47 verso, 47 recto)

(i) ·· व्यूह पार्थं हेहयकों ध्नत

सायकैश्चैव ф पट्टि स्वपाद दल षोडशैं ।

अ···न्या चतस्रा वै हता तेन महात्मवान् ॥

शराणां च परीमाणं···विशारद ॥

शि १ १	१६ १ १ ४ १ २	४ अ० छे० २१८७० १ १	फलं शरा २६२४४००

अन्या ई प्रमाणम्

(ii) सूत्रम् ॥ एको रथो गज···

(i) ··· ··· ···

··· ··· विचक्षणः

चमूस्तु पृतनास्तिस्रस्तिस्रश्चम्···

अनीकीनि दशगुणां आहु 'आहुरक्षोहनी बुधः ॥

अक्षोहि···

र० १	एष	३ ३ ३ ३ ३ ३ ३ १० गु०	
ग० १	पति	१ १ १ १ १ १ १ १	
न० २५	गुणिता जात:	रथ	२१८७०
तु० ३		गज	२१८७०
		नर	१०९३५०
		हय	६५६१०
			(२१८७००)

एष अक्षोहिणी प्रमाणम् ॥

(ii) उदा० ॥ कश्चिद् राजकुमार शत्रुदम ।

M 8.

(48 recto, 48 verso)

कि	दि०	र० १···	व० ३ / १ / १ छे०
		य० १ ३* / १ / ५ भा० क्ष- ८०* / -य १ / ३	भा० ३ / १ / १२*
		क० १ ६* / १ / ४ भा०	
		पा० १ / ४*	दि० १ / ३०*
		शे० १ / ३	

छेदम् ४८०* रक्ति-पल··· गुणितं जातं ४१९९४२···३६ पल

११५२००

तो० ८ पले तो० ३ तोलेनास्ति धा० १२* धा० ७ धाने नास्ति अ० ४* अं० २

(i) | फलं भा० २···एनास्ति
पल २००० भा० । प० २७० ॥···तो० ८

छे० ८* / धा० २ / छे० १२ / गुं० ३ / छे० ५*	तोल-पल तो० ६ तोले नास्ति धाण १२ धा० ८ ॥
य० २ ३* / १ १ / ५ भा	

(ii) यदि दिनमेकेन एष दत्तं तद् द्वादश वर्षेण···

दि० १	२१६ भा०	वर्षे १२ ३···	फलं भार ६३···
१	२७० प०	१ १	
	२०००* छे०		
	६ तो०		
	८* छे०		
	८ धा०		
	१२ छे०		

M 9

(49 verso, 49 recto)

···रक्ति क्षय···पञ्च गुणम्

दिवसा विंशतिकं किं शुंद्यति महां···वद निश्चयम्

१	तो० ३		क्षय ४+	व० २५	छे० ३६०
१	मा० २		६०*	म० ५	१
	१२*		शि० ४	१२	
	अं० ३		८*	दि० २०	
	४*			३०	
	य० ३				
	४*				
	क० १	६*			
	पा० १	१			
	४*	४			
	मू० १				
	४*				

६२३२१	क्षयं शोध्य···	६०८८१	अधच्छेदं २०००
१९२०		१६२००	

सर्वगुणितं	५५८२७८७७०	···७ तोले पलम्···
	१६२००	

··· ··· ···

(i) य० ३ यवनास्तिक० ६
१ द० कलानास्तिपा
४
पादनास्ति मूदृ० ४ पामू० मू० २ ॥

(ii) उदाहरणम् ॥

···शूख्यैयंजन्ति देवी प्रतिमह्निकेचित् ददामि देव्या···कञ्चः क्रीत्वा दीनारशतानिचत्वारित धानकाण्डिका रक्तिकायवाकलापादमूदृका च् । एतद् मूल्यं वद मे तत्र ···स्य किम्

१ तो १२* | मू ४०० | धा १ | फलं दी ५० दीनार नास्ति धाणे
१ १ | १ | अं १
४* | १२* धानका १० धाणे नास्ति अं ४*
र १ १* भा
१
४
| अं १ ॥
य १ ३* भा
१
५
क १ ६* भा
१
४
पा १
४ *
मू १
४ *

M 10.

(55 recto, 55 verso)

(i) | तो० १ | व० ५ | तो० १ | फ० व० ६ ··· शे० | ६ | गुणितम्
      १ | १ | धा० १ | | १० |
            ३ | १२* | | ७२२७ |
               | अं० १ | | १२०० |
               | ४* |
| र० १ १* भा०
        १
        ४
| य० १ ३* भा०
        १
        ५
| सि० १ २* भा०
        १
        २
| क० १ २* भा०
        १
        २
| पा० १
      ४*
| मू० १
      ४

अथ षड् द्रम्मको ··· ज्जारद्विधानकैद्रमं श ··· विंशतिपाला हृतै धानका ।

अस्यैव स्कन्ध—

(ii) | तो० १ | व० ५ | तो० १ धा० १ १* अं० १ १* र० १ १* य० १ १*
      १ | १ | १   १ १२   १ ४८   १ ६०   १ १६२
            ३ |

सि० १ १* क० १ १* पा० १ १* मू० १ १
    १ ४८०   १ १२०   १ ४८००   १ १६२००

न्यास	तो० १ १	३५ १	१ तो० १ २ १ १* मा० १ ६ २ १ १* अं० १ २ २ १ १* य० १ २ २	फ० द्रं० ५८ शे०	३१ १२८

पुनान्यम्

तो० १ १	३५ १	१ १ १ १ २ २	१* १२	१ १ २	१* ४८	१ १ २	१* १६२	फलं ५८ शे०	३१ १२८

M 11.

(44 verso, 44 recto)

....

नीवी सप्त-शतानां क*X* कालमार्जन भक्षणे ॥

न्यास स्थापनं क्रियते

आ० १ दि० १ भा० १ १ २ ३	भ० ८ दि० ५ भा० १ प० ३२ भा० शू० २ ३६ भा० १ १ १ १ १ १ १ ३ २ ४	१ १ १ ३६०

भाण्डा ७०० १

व्यय राशि | २२३
१४४ | आय राशि | २८०
६१ | एतत् कालेनार्जन भक्

दि १ १	२२३ १४४	२८० ६१	ऊर्ध्वच्छेदं ३६ फलं ...... २४५६ शे १ ६१

एष व्यये ॥

...	व० ४ मा० ७ दि० २ शे०	२८ ६१	

अथ आय	द० १ १ २	१ १ ३	२८० ६१

(i)	२५५६ १ ६१	दि १ १	——	२२३ १४४	एष व्यय प्रमाणम्

(ii) उदा० ॥ एक दशार्घमुत्पति स त्रिभाग दिन द्वयेत्

पूजार्थं स त्रिभागञ्च त्रयोदश ... तताश्चयेत्

षष्ट भाग दिना त्रीणि बासुदेवस्य चार्चयेत्

पादोन त्रयोदशानाञ्चाष्ट सार्ध दिनानि चेत् ॥

ब्राह्मणा भोजने दद्या परलोकहितार्थिनः

स त्रिभागं ...... ज्जारं स पञ्च भाग दिन त्रयेत्

प०

सार्धं सार्धं दिने

M 12.

(43 recto, 43 verso)

...., आरयेत्

......सार्ध द्वादशयेवात्र भोजने मद्यमत्तमेत्

स त्रिभाग त्रयस्त्रिंशै दिनैर्द्वाणिज्यकस्य तु ।

भाण्डारे द्वादशशत वजाराणां स्थितास्य वै ।

एषा व्यय समुत्पत्तौ कः X कालं ब्रूहि पण्डित ॥

करण विधानेन द्वादश शतस्य भाण्डारे स्थितता

१० २ भा० १३ ३ भा० १२ ८भा०१ ३ भा० १ १ भा० १ ५ भा० २ १३ भा०
१ १   १ १   १ ? १ १   २ १   १ १ [१ १ १
२ ३   ३ ८   ४+२ ३ ५   २   ३ [४ ४ ४

१२ ३३ भा० | १ १ | भाण्डा १२०० गुणितानि
१ १ | १ ३६० | १
२ ३

| २ १० ८०० | अर्धच्छेदं ३६० दिव......तेन सह ॥ आय-पिण्डं
| १ १ ७२७ |
| ३ २

| २९८२ | ... अधुना व्यय पिण्डं | दि० १ | १८०७ | ८०० |
| ४८६ | | १ | २४० | ७२७ |
| ७२७ |

ऊर्धच्छेदं ३६० फलं दिव | २९८२ | पुन | ८०० | २९८२ | १
| ४८६ | | ७२७ | ४८६ | १
| ७२७ | | | ७२७ |

अधच्छेदं ३६० दि......फलं प्रतिदिन...... | १८०७ | एवं सर्व त्रि-राशिकेन.
| २४० |

उदा०

M 13.

(42 verso, 42 recto)

आर्धं युक्ते त्रयोदश सार्धं भवति

४० भा० | १६० | १३ |...एषां च्छेदं कृता जाता एकेन
१ | १ | १ |
| | २

.. सार्धं त्रयोदशभि किमिति | १ | ४ | २७ | फ० ५४ एषां...
| १ | १ | २ |

...एकेन लब्ध चत्वारिष् पड्भि संपद्यते कथ | १ ४ |
| १ |

...एको लभति चत्वारि संसर्घम्य तु किं भवेत् ..

(i) जाता ५४ । षड्त्रि २४ । ... १२ । अर्धा १८ । एकत्रम् ५४ ।।

ए... त्रैराशिक करण प्रत्येक मूल्य विधि ।

(ii) अपरं वक्ष्यामि । विंशानों दिव ' किं प्रथमे खन्धकेषु यो-

भिलिखित । अपस्य प्रष्णा विधि

२०	१ १
१	१ ३
	२

गुणये । गुणिता

जाता

२०	३	१
१	२	३

छेदं

२०	१	१
१	२	१

भागे...जातं फलं रू

१० ।। एष विंशानां दिव... भवति । अत्र उपरिमाश्खन्धकस्य एष

गुणाकारं भवति ।...

M 14.

(50 verso., 50 recto.)

(i) द्रम्मे त्रपुस शतं लब्धं अर्धेण लभ्यते X कति ।

एक राशिस्तु कलना गणित प्रक्रिया कुरु:

१ द्रम्मे	फलं ५०
१०० द्र पुसा	
१	
२	

(ii) अपरं उदा० ।। साधं द्वये त्रयसाधं दित्रर्धे लभ्यतेXकति

२
१
२
१
२
१
२
१

सूत्रम् ।। अर्धेनोपरि संगुण्य ··· वर्ध क्रमेण च ।

अर्धेन ऊर्धं गुणये म ··· पञ्च संगुणं ।

भाजये लब्ध पण्यं

········· वशिष्ट पुत्र

सिकस्यार्थे पुत्र पौत्र उपयोग्यं भवतुः

लिखितं छज्जक पुत्र गणकराज ब्राह्मणेन ।

सर्वेषामेव शास्त्राणां गणितं मूर्ध्नि तिष्ठति ।

आद्यावसाने संसारे उत्पन्न ·· महत्

पश्चा सृष्टि तदा कर्तुं शिवेन परमात्मना

···यद्यञ्चमुत्पन्नं गणितं सव्य कारणम् ।

यच्··· ··· ···

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