Vedic Mathematics in 20th century

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The Vedic Mathematics as initiated by Swami Bhārati Krishna Tīrtha was an exciting new event in 20th century India. It promised to give easier methods for working with all branches of mathematics. The methods are also promised to be faster, concise and possessing a natural elegance. It is no wonder that it has caught the fancy of the common man as well as the political machine.
As a result, there are enthusiastic people actively seeking to give it the position of the prime pedagogical tool all over the world.
We will examine how much Vedic it is and what amount of mathematics it contains and what role it can play for the advancement of Mathematics.

At the turn of the nineteenth century in 1884 a child named Venkatraman was born in the Madras Province (now Tamilnadu) who went on to become an exceptionally gifted student. He was well versed in Western as well as Indian sciences. At the age of 23, he passed M.A. examination of the American College of Science in Rochester, securing the highest honors in all the seven subjects. His life was leading in the direction of illustrious public service when in 1911 he took a sharp turn to Vedanta and ascetic life and after eight years of study and hard work, earned the title Bhāratī Kṛṣṇa Tīrtha after formal initiation. He spent his remaining life lecturing all over the world, promoting Hindu spirituality, world peace and a new way of doing Mathematics called Vedic Mathematics.

To most of us, Vedic refers to the entities related to the four Vedas, the Vedāñgas as well as the various philosophical treatises known as the upaniṣats. Usually this does not even include the smṛti granthas though they are well respected and does not include the purāṇās. Swamiji proposes a much wider meaning. He wishes to include later treatises like dhanurveda, gāndharvaveda etc. and puts forth the opinion that true Vedas must include all knowledge needed by mankind for spiritual as well as "secular" or "worldly" matters, essential for the *achievement of all-round, complete and perfect success in all conceivable directions...* In effect, his contention is, that any such useful knowledge must be found somewhere in the Vedas as we know them. He reports that he had a strong personal desire to prove this in relation to Mathematics and sciences and that it was further fired up by the *contemptuous or, at least patronizing attitude adopted by the Orientalists, Indologists...*

Thus proving that the view of Mathematics developed by him after many years of hard work was rooted in a Vedic source was a very clear priority, nay, perhaps even an axiom for him! This is more or less like the philosophy of Gītā which declares that the best among each worldly species is the part corresponding to the Lord Kṛṣṇa. He reports being agreeably astonished and intensely gratified to find ... ultra-easy Vedic Sūtras contained in the Pariśiṣṭa of the Atharvaveda. It is unfortunate that these are nowhere to be found by the rest of us and the general Editor V.S. Agrawala has to concede (in his note in the book Vedic Mathematics by Swamiji) that this Pariśiṣṭa should simply be regarded as a new one attributed to Swamiji himself, despite Swamiji's own assertion and conviction to the contrary. He proposes that whatever is contained here stands on its own merits and is presented as such to the Mathematical world.

So, let us ignore for now, the problem of looking for a missing version of Atharvaveda and proceed to examine the contents of the Vedic Mathematics, which I propose could be and should be aptly called Tīrtha-Gaṇita or TG for short.

Swamiji explains at great length that the current system of working out Mathematical problems is needlessly tedious and cumbrous and that TG gives fast and easy methods to do the same problems with fewer steps and amazing ease. We note that many of the methods of mathematical calculation, especially in arithmetic, actually come from the old Mathematical wizards like Āryabhaṭa, Brahmagupta, Bhāskara and so on, who probably had studied what they knew as Vedic mathematics. Thus, if TG was lost at some historic date, it was really completely lost; it has no known Āchāryas, no known results, not even a passing reference to it is to be found anywhere, until Swamiji found it. I should add that there were many other clever calculation techniques being proposed by these old masters and what we describe today as the traditional methods of calculation were probably considered the best among the choices and hence survived.

We now examine the topics in TG one by one and their value to the Mathematical world.

1. A large part of the book (declared to be the first in a series of 16) is devoted to the basic techniques of adding, multiplying and division of integers. Most techniques are based on the following idea of Swamiji:
   Every decimal number can be thought of as a polynomial in 10 with coefficients from 0, 1,..., 9. For example 3407 is simply $(3)10^3 + (4)10^2 + (7)$.
   Now, working with polynomials is actually a harder task for a student, however, polynomials have one distinct advantage. We can easily identify and collect the right terms to find the entry in the desired spot (i.e. a decimal place). As a result, Swamiji manages to give recipes to calculate the answers in a single line by doing the intermediate calculations mentally.
   When one or more of the terms being operated on has a particularly special form, Swamiji's TG prescribes (or suggests) alternative shortcuts. It is true that the shortcuts as described will indeed give the answer in a much shorter time than the traditional technique, but the time spent on thinking of the right method or sūtra to use is nowhere mentioned or estimated. Thus, with training, a student can get good at a limited set of problems; but may not know how to do the most general case, if it does not fit one of the standard cases of TG.
2. The book discusses working with rational functions. Special techniques are proposed for factoring quadratic polynomials, but the special cases of guessing the factorization are the main point of discussion.
   There is an extensive discussion of partial fractions when the factors of the

denominator are linear with or without multiplicity. There are tricks to guess the corresponding numerators when possible and calculate with minimal calculation if needed. Most of these tricks are well known and taught in regular colleges, without the colorful sūtra names, of course. In spite of these tricks, when one has two or more quadratic factors (irreducible over reals) one has to resort to the traditional long method and hence the modern mathematics makes a point of teaching the general method. Also, for a proper theory of rational functions, one needs to establish the existence of partial fraction in the most general case!

In TG, methods are suggested for finding partial fractions which avoid the analysis of the most general case, because the general case is rarely "a neat pretty package". Elsewhere, we find Swamiji declaring certain techniques as complete and valid but not satisfactory only because they are not sufficiently compact!

Only special polynomials of higher degree are discussed, especially the ones which are neatly factored over integers.

Swamiji promises more details in future volumes but does not give any sample of promised results. Alas, we shall never know how he proposed to treat the big theorem of modern mathematics declaring general polynomials of degree 5 or more to be unsolvable by root extraction formulas. That wonderful theory (the Galois Theory) will have to live in the non Vedic world for now.

There is a brief beginning of linear algebra, but it is restricted to a few "lucky" equation systems for now. There is a discussion of integration of polynomials where the ekādhika sūtra is used to explain integration of x^n to $\frac{x^{n+1}}{n+1}$. There is nothing new in the method or theory here and the use of the sūtra is not significant.

3. There is some discussion of elementary analytic geometry, again with the same methodology. For convenient problems, fast methods of resolution are given. There is essentially no discussion of what happens to the general case. The fact that with TG you need to change your method of solution by looking at each problem (the vilokana) is described as the freedom of choice afforded by the TG, rather than a deficiency.

   There is a claim made that the terms "Arabic numerals, Pythagorean Theorem and Cartesian coordinates" are historical misnomers. While the first is true and the second is debatable, the third one is almost certainly false!

   There are proofs given of some of the well known theorems of Pythagoras and Apollonius.

   By the way, in case of Pythagoras, it is not clear that he gave a proof, and hence some suggest that the earlier Shulba sūtras might have a valid claim on the theorem for they certainly asserted it and were clearly using it in their constructions with conviction. Actual proofs were late in coming and were not commonplace in the Indian works anyway.

   The proofs of the theorems presented by Swamiji are not new and the proofs of Pythagoras Theorem have appeared elsewhere in the world (China, among others). There is no evident use of the famous sūtras from TG in the proof and the analytic geometry is not used either.

   Also, I wonder why there is no "new" theorem in the whole topic. If TG is indeed ancient and more efficient, somebody should have discovered a theorem not known to the Greeks!

4. The reader may get the feeling that I found nothing useful or interesting in the book. That is not entirely true and I now proceed to describe some of the achievements of the book.
   The method proposed by Swamiji for calculating the decimal expansion of a fraction has all the qualities of an ideal TG result as well as a mathematical theorem in the traditional sense. It is a very efficient method of getting the successive digits of the decimal expansion (which of course repeat in block after a suitable initial piece). The method illustrates the full power of realizing the decimal number as a generalized polynomial in 10 with negative powers allowed. The method shows good command over calculations modulo the denominator. It is certainly new and fairly easy to execute.
   The proof is easy, once you think of the method and the proof can be taught to young students or even discovered by the talented kids, once they know the result.
5. An even more interesting result is a universal test of divisibility. This is so easy that I can even write it down in a short paragraph.

   Suppose you have a number like 23 called a divisor and you wish to see if it divides a certain number, say 7211, called the target.
   First you find a certain multiplier thus:
   Find some multiple of 23 which ends in 9, say 69. Then you get that (7) (10) = 1 + (multiple of 23, namely (3) (23)).
   This 7 is simply read off by the ekādhika from the 6 (part before 9).
   The procedure is to write your number as 10 a+b (here a=721 and b = 1).
   Then (7) (10a+b) = (7) (10)a+7b = a+7b plus some multiples of 23.
6. Thus we get the result that the given number (10a+b) is divisible by 23 if and only if (a+7b) is. This new number (a+7b)

   The main point is that this gives a successive reduction.
   Say: 7211 $\rightarrow$ 721+7=728 $\rightarrow$ 72+56 = 128 $\rightarrow$ 12+56 $\rightarrow$ 68 which is not divisible!
   So, 7211 is not divisible by 23.

   The calculation of the multiplier can be easily described in complete generality and Swamiji actually gives a detailed analysis including a generalization to handling a multi-digit reduction.
7. ***Challenge:*** If someone could find a similar easy technique for finding the actual remainder of one number modulo another, it could have far reaching consequences in number theory. The method given by Swamiji is a clever twist on the usual divisibility tests in elementary number theory books but accomplishes far more, except the final answer is not the same as the original target number modulo the divisor.

8. ***Query:*** I have on good authority that this method without mention of the sūtras was being taught in the 60s in Tamilnadu high schools. I would very much like to

know if it was a consequence of the TG book or was there an independent source. It certainly was not being taught in our high schools in Maharashtra.

9. I finally come to a very curious and interesting remark in the book. Swamiji claims to give a verse which gives the value of π the famous ratio of the circumference to the diameter correct to 31 decimal places. Here is the verse in the well known kaṭapayādi system

   गोपीभाग्यमधुव्रात-श्रृङ्गिशोदधिसंधिग।

   खलजीवितखाताव गलहारारसंधर॥

   This gives the value (after inserting the decimal at the second spot):
   3.1415 9265 3589 7932 3846 2643 3832 792
10. The curious thing is that the last digit is wrong, after 79 the correct expansion continues 50288.
   Swamiji either does not know the error or chooses to ignore it; for any correction would need the last letter of the verse to end in one of ङ ण भ श followed by a vowel.
   What is even more curious is that Swamiji claims that the verse has a "self-contained-master-key" to generate any further number of digits!
   Why, oh why did he not write a book about the key, rather than the rest of TG? I would guess that had that happened, his accomplishment could have eclipsed the fame of the other great soul from Tamilnadu born three years after Swamiji and who, after a short meteoric life full of mathematical excellence passed away in 1920 at only 33 years of age. I speak of S. Ramanujan, of course, whose results and conjectures are still being researched and absorbed by the mathematical community.

11. **Conclusion:** It is stated that the book by Swamiji as it stands, aims to prepare a student for efficiently working out the problems which typically occur in the textbooks. Swamiji has great reverence for the decimal system (in one place he laments the fact that people still continue to use "vulgar" fractions) and wishes to convert all numbers to the decimal system as well as use their peculiar properties based on the base of 10. While the decimal system might be of special significance to scientists and engineers, Mathematics has no special use for it. Indeed, the natural number theoretic properties of fractions are obscured if you convert them to their decimal equivalents.
   Mathematically, the only interesting parts of the book are the universal divisibility tests, and the topic of finding the decimal expansions of fractions (actually the method, not the result).
   People, who promote and actively work with Swamiji's Vedic Mathematics, claim that it is good for the young students, giving them a renewed confidence in and love of mathematics. If that is indeed the case, I have no argument against it, indeed, I am all for it!
   However, it should not lead to the false notion that "Mathematics means fast

resolution of selected textbook problems". True advances in Mathematics come from long hard work on significant problems. Along the way, you may do a lot of calculations. These calculations are but a tool to help the intuition. With the advent of technology and computer science, human speed of calculations is becoming less and less important. So, if you wish to develop new mathematics, it is far better to engage in a creative mathematical thought process rather than spend time doing routine calculations.

My more serious concern is this. With the "Vedic mystic" attached to it, I can envision the possibility of people following the TG and its methods with blind faith. I can see talented bright people developing more methods to solve special cases of more text book problems. Moreover, the available creative talent as well as the motivation to investigate mathematical truths may diminish or vanish as a consequence. I can visualize mathematicians spending all their talent in trying to find the Vedic origin of theorems rather than attempting to prove new ones. I have already seen at least one pseudo proof of the Twin Prime conjecture by spurious application of the TG sūtras!

Nothing would please me better than being proved completely wrong in all these visions!