eastern tropical Pacific and Antarctica peaked during each of the last two glacial terminations (*28*), consistent with the timing of enhanced EPR hydrothermal activity.

Isolating a mechanistic linkage between ridge magmatism and glacial terminations will require a suite of detailed proxy records from multiple ridges that are sensitive to mantle carbon and geothermal inputs, as well as modeling studies of their influence in the ocean interior. The EPR results establish the timing of hydrothermal anomalies, an essential prerequisite for determining whether ridge magmatism can act as a negative feedback on ice-sheet size. The data presented here demonstrate that EPR hydrothermal output increased after the two largest glacial maxima of the past 200,000 years, implicating mid-ocean ridge magmatism in glacial terminations.

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SUPPLEMENTARY MATERIALS

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HISTORY OF SCIENCE

Ancient Babylonian astronomers calculated Jupiter's position from the area under a time-velocity graph

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The idea of computing a body's displacement as an area in time-velocity space is usually traced back to 14th-century Europe. I show that in four ancient Babylonian cuneiform tablets, Jupiter's displacement along the ecliptic is computed as the area of a trapezoidal figure obtained by drawing its daily displacement against time. This interpretation is prompted by a newly discovered tablet on which the same computation is presented in an equivalent arithmetical formulation. The tablets date from 350 to 50 BCE. The trapezoid procedures offer the first evidence for the use of geometrical methods in Babylonian mathematical astronomy, which was thus far viewed as operating exclusively with arithmetical concepts.

The so-called trapezoid procedures examined in this paper have long puzzled historians of Babylonian astronomy. They belong to the corpus of Babylonian mathematical astronomy, which comprises about 450 tablets from Babylon and Uruk dating between 400 and 50 BCE. Approximately 340 of these tablets are tables with computed planetary or lunar data arranged in rows and columns (*1*). The remaining 110 tablets are procedure texts with computational instructions (*2*), mostly aimed at computing or verifying the tables. In all of these texts the zodiac, invented in Babylonia near the end of the fifth century BCE (*3*), is used as a coordinate system for computing celestial positions. The underlying algorithms are structured as branching chains of arithmetical operations (additions, subtractions, and multiplications) that can be represented as flow charts (*2*). Geometrical concepts are conspicuously absent from these texts, whereas they are very common in the Babylonian mathematical corpus (*4–7*). Currently four tablets, most likely written in Babylon between 350 and 50 BCE, are known to preserve portions of a trapezoid procedure (*8*). Of the four procedures, here labeled B to E (figs. S1 to S4), one (B) preserves a mention of Jupiter and three (B, C, E) are embedded in compendia of procedures dealing exclusively with Jupiter. The previously unpublished text D probably belongs to a similar compendium for Jupiter. In spite of these indications of a connection with Jupiter, their astronomical significance was previously not acknowledged or understood (*1*, *2*, *6*).

A recently discovered tablet containing an unpublished procedure text, here labeled text A (Fig. 1), sheds new light on the trapezoid procedures. Text A most likely originates from the same period and location (Babylon) as texts B to E (*8*). It contains a nearly complete set of instructions for Jupiter's motion along the ecliptic in accordance with the so-called scheme X.S$_1$ (*2*). Before the discovery of text A, this scheme was too fragmentarily known for identifying its connection with the trapezoid procedures. Covering one complete synodic cycle, scheme X.S$_1$ begins with Jupiter's heliacal rising (first visible rising at dawn), continuing with its first station (beginning of apparent retrograde motion), acronychal rising (last visible rising at dusk), second station (end of retrograde motion), and heliacal setting (last visible setting at dusk) (*2*). Scheme X.S$_1$ and the four trapezoid procedures are here shown to contain or imply mathematically equivalent descriptions of Jupiter's motion during the first 60 days after its first appearance. Whereas scheme X.S$_1$ employs a purely arithmetical terminology, the trapezoid procedures operate with geometrical entities.

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In text A, Jupiter's motion along the ecliptic is described in terms of its daily displacement (modern symbol: v) expressed in °/d (degrees/day) and its total displacement (S) expressed in degrees. A crucial new insight about scheme X.S$_1$ provided by text A concerns its use of piecewise linearly changing values for v. Although not formulated explicitly, this linear dependence on time is clearly implied (*8*). Jupiter's motion along the ecliptic is described for two consecutive intervals of 60 days between its first appearance and its first station. For each interval, initial and final values of v are provided. Note that Babylonian astronomy employs a sexagesimal; i.e., base-60 place-value system in which numbers are represented as sequences of digits between 0 and 59, each associated with a power of 60 that decreases in the right direction. In the commonly used modern notation for these numbers, all digits are separated by commas, except for the digit pertaining to 60°, which is separated from the next one pertaining to 60^{-1} by a semicolon (;), the analog of our decimal point. For the first interval of 60 days, v_0 = 0;12°/d (=12/60) and v_{60} = 0;9,30°/d (=9/60 + 30/60^2). Their sum is multiplied by 0;30 (=1/2), resulting in a mean value $(v_0 + v_{60})/2$ = 0;10,45°/d, which is multiplied by 1,0 (=60) days, resulting in a total displacement $S = 1,0\bullet(v_0 + v_{60})/2$ = 10;45°. For the second interval, v_{60} = 0;9,30°/d and v_{120} = 0;1,30°/d (=1/60 + 30/60^2), leading to $(v_{60} + v_{120})/2$ = 0;5,30°/d and S = 5;30°. The sum of the total displacements, 10;45° + 5;30° = 16;15°, is declared to be the total distance by which Jupiter proceeds along the ecliptic in 120 days. In other words, the ecliptic longitude of Jupiter after 60 and 120 days is computed as $\lambda_{60} = \lambda_0$ + 10;45° and $\lambda_{120} = \lambda_0$ + 16;15°, respectively.

Text A does not describe how v varies from day to day, but of the three forms of time dependence of v that are attested in Babylonian planetary texts—piecewise constant, linear, or quadratic in each time interval (*2*, *9*)—only the linear one comes into question. If v were piecewise constant, then S should equal $60\bullet v$ for each interval. If v were piecewise quadratic, then $S = 60\bullet(v_0 + v_{60})/2$ can only be some rough approximation. That would be unexpected, since other tablets imply that some Babylonian scholars in this period were familiar with the exact algorithm for summing a quadratic series (*9*, *10*). By contrast, the values of S computed in text A are exact if one assumes that v changes linearly in each interval. It follows that in scheme X.S$_1$, v decreases linearly from 0;12°/d to 0;9,30°/d between day 0 and day 60, and from 0;9,30°/d to 0;1,30°/d between day 60 and day 120.

This new reconstruction of the first 120 days of scheme X.S$_1$ results in trapezoidal figures if v is plotted against time in a modern fashion (Fig. 2). It is important to note that text A itself does not contain or imply a geometrical representation. However, it turns out to be explicitly formulated in the trapezoid procedures, texts B to E (figs. S1 to S4). Although their formulation differs in details, at least three of them (B to D) consist of the same two parts, I and II.

In part I, Jupiter's total displacement for the first 60 days of scheme X.S$_1$ is computed. A corresponding introductory statement mentioning Jupiter and the measures of the trapezoid is partly preserved in texts B and C, and perhaps in text E (*8*). The number 10;45, referred to as the "area" of the trapezoid (B, C), is then added to the "position of appearance" (B, C, D), the technical term for Jupiter's ecliptical longitude at first appearance, i.e., $\lambda_{60} = \lambda_0$ + 10;45°. Texts B and C partly preserve the computation of 10;45 as the area of the trapezoid through a series of steps equivalent to the computations in text A. Its "large side" and "small side," v_0 = 0;12°/d and v_{60} = 0;9,30°/d, are averaged, $(v_0 + v_{60})/2$ = 0;10,45°/d, which is then multiplied by 60 days, the width of the trapezoid, resulting in 10;45°. The latter operation is partly preserved in text C and can be restored in text B.

Part II, partly preserved in texts B, D, and E, is concerned with the time in which Jupiter reaches a position referred to by a term tentatively translated as the "crossing" (*8*). It is now clear that this denotes a point on the ecliptic, say λ_c, located halfway between λ_0 and λ_{60}, i.e., $\lambda_c = \lambda_0$ + 10;45°/2. This interpretation is consistent with a statement, preserved only in text B, according to which the "crossing" is located in the middle of Jupiter's "path," readily interpreted as a reference to the ecliptical segment from λ_0 to λ_{60}. Texts B and D also preserve the following statement that

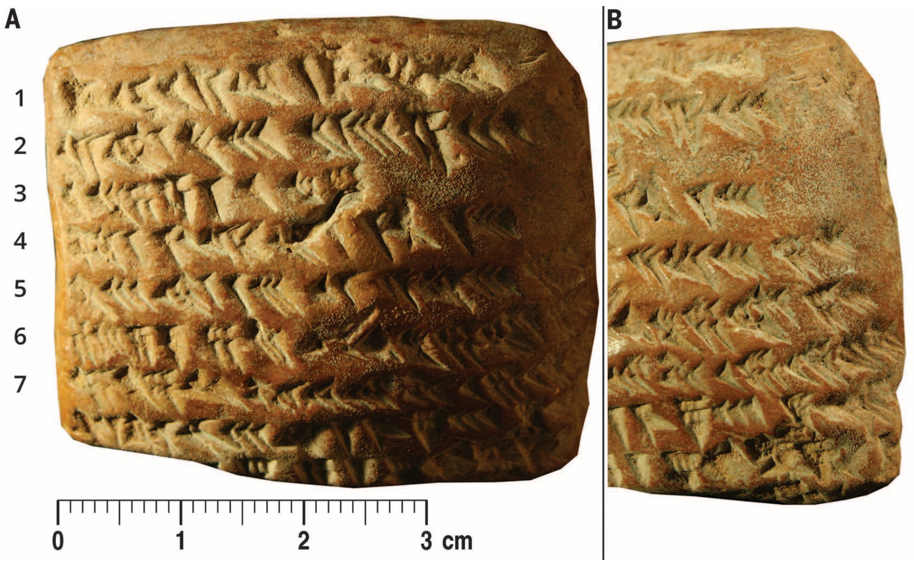

Fig. 1. Photograph of text A (lines 1 to 7). (**A**) Full image. (**B**) Partial image of the right side taken under different lighting conditions.

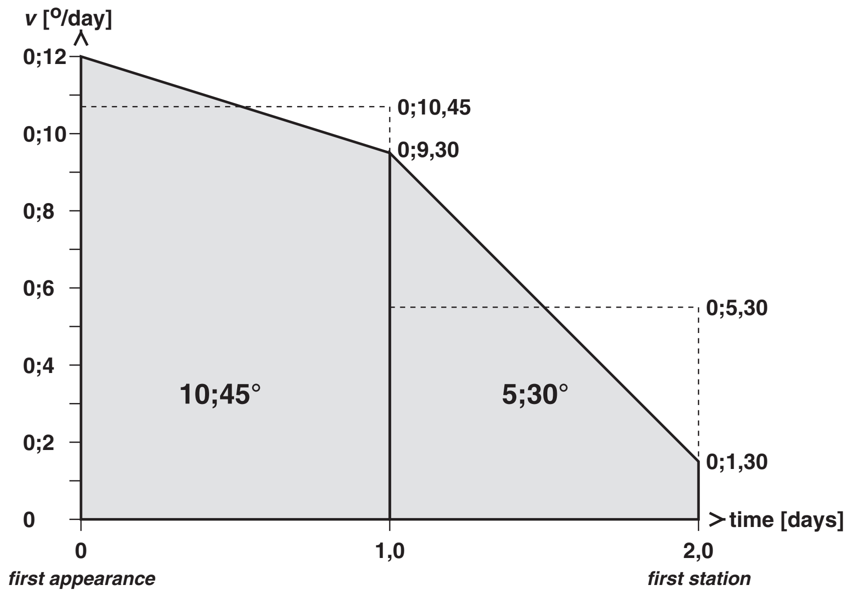

Fig. 2. Time-velocity graph of Jupiter's motion. Daily displacement along the ecliptic (v) between Jupiter's first appearance (day 0) and its first station (day 120) as a function of time according to scheme X.S$_1$ as inferred from text A. All numbers and axis labels are in sexagesimal place-value notation. The areas of the trapezoids, 10;45° and 5;30°, each represent Jupiter's total displacement during one interval of 60 days.

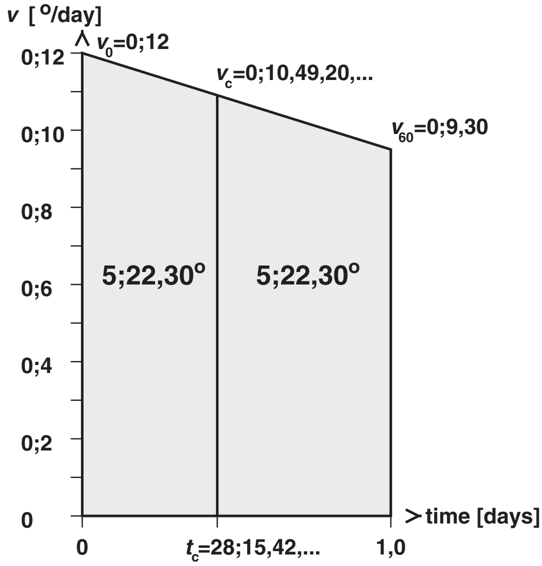

Fig. 3. Partitioning the trapezoid for days 0 to 60. The time at which Jupiter reaches the "crossing," t_c, where it has covered the distance 5;22,30° = 10;45°/2, is computed geometrically by dividing the trapezoid for days 0 to 60 into two smaller trapezoids of equal area. In text E, v_c is rounded to 0;10,50°/d, resulting in t_c = 28 d, S_1 = 5;19,40°, t_2 = 32 d, and S_2 = 5;25,20°.

precedes the solution procedure: "Concerning this 10;45, you see when it is halved." The time in which Jupiter reaches λ_c, say t_c, is then computed by the following geometrical method: The trapezoid for days 0 to 60 is divided into two smaller trapezoids of equal area (Fig. 3). In order to achieve this, the Babylonian astronomers applied a partition procedure that is well-attested in Old Babylonian (2000 to 1800 BCE) mathematics (*5*, *6*). In modern terms, it can be formulated as follows: If v_0 and v_{60} are the parallel sides of a trapezoid, then the intermediate parallel that divides it into two trapezoids of equal area has a height $v_c = [(v_0^2 + v_{60}^2)/2]^{1/2}$. In the present case, v_c denotes Jupiter's daily displacement when it is at the "crossing." This expression follows from equating the areas of the partial trapezoids, $S_1 = t_c \bullet (v_0 + v_c)/2 = S_2 = t_2 \bullet (v_c + v_{60})/2$, where t_c and t_2 are the widths of these trapezoids, and using $t_c = t \bullet (v_0 - v_c)/(v_0 - v_{60})$, where $t = t_c + t_2$ is the width of the original trapezoid (*6*, *10*). Inserting v_0 = 0;12°/d, v_{60} = 0;9,30°/d, and t = 1,0 d, we obtain $v_c = [(0;2,24 + 0;1,30,15)/2]^{1/2} = (0;1,57,7,30)^{1/2}$ = 0;10,49,20,44,58,...°/d, t_c = 28;15,42,0,48,...d, and t_2 = 31;44,17,59,12,...d. The computation of v_c is partly preserved in text D up to the addition 0;2,24 + 0;1,30,15 (*8*). In text B, the related quantity $u^2 = (v_0^2 - v_{60}^2)/2$ = (0;2,24 – 0;1,30,15)/2 = 0;0,26,52,30 is computed. This was most likely followed by another step in which v_c was computed using $v_c^2 = v_0^2 - u^2$. Whereas all known Old Babylonian examples of the partition algorithm concern trapezoids for which v_c, v_0, and v_{60} are terminating sexagesimal numbers (*6*), the present solution does not terminate in the sexagesimal system. Hence, texts B to E can only have offered rounded results for v_c and t_c. Nothing remains of this in texts B to D, but text E partly preserves a computation involving 0;10,50, which is, most plausibly, an approximation of v_c. This interpretation is confirmed by the fact that text E also mentions the value t_c = 28 d and, very likely, t_2 = 32 d, both in exact agreement with $t_c = 60 \bullet (v_0 - v_c)/(v_0 - v_{60})$ and $t_2 = 60 - t_c$ if one approximates v_c = 0;10,50°/d. By rounding v_c, only an approximately equal partition of the trapezoid is achieved.

Also partly preserved in text E is a computation of the area of the second partial trapezoid, using the same method as before, leading to $S_2 = t_2 \bullet (v_c + v_{60})/2$, where t_2 = 32 days, v_c = 0;10,50°/d, and v_{60} = 0;9,30°/d. The value of S_2 is broken away but can be restored as 5;25,20°. The probable purpose of this computation was to verify the solution for v_c, as is done in the Old Babylonian mathematical text UET 5, 858 (*5*, *11*). The analogous computation of the area of the first partial trapezoid, which can be reconstructed as $S_1 = t_c \bullet (v_0 + v_c)/2$ = 5;19,40°, is not preserved. Neither of these values equals 5;22,30° = $S/2$ as they ideally should (Fig. 3), a direct consequence of the rounding of v_c to 0;10,50°/d. At most two more lines are partly preserved in texts B, D, and E, but they are too fragmentary for an interpretation.

The evidence presented here demonstrates that Babylonian astronomers construed Jupiter's displacement along the ecliptic during the first 60 days after its first appearance as the area of a trapezoid in time-velocity space. Moreover, they computed the time when Jupiter covers half this distance by partitioning the trapezoid into two smaller ones of ideally equal area. These computations predate the use of similar techniques by medieval European scholars by at least 14 centuries. The "Oxford calculators" of the 14th century CE, who were centered at Merton College, Oxford, are credited with formulating the "Mertonian mean speed theorem" for the distance traveled by a uniformly accelerating body, corresponding to the modern formula $s = t \bullet (v_0 + v_1)/2$, where v_0 and v_1 are the initial and final velocities (*12*, *13*). In the same century Nicole Oresme, in Paris, devised graphical methods that enabled him to prove this relation by computing s as the area of a trapezoid of width t and heights v_0 and v_1 (*12*). Part I of the Babylonian trapezoid procedures can be viewed as a concrete example of the same computation. They also show that Babylonian astronomers did, at least occasionally, use geometrical methods for computing planetary positions. Ancient Greek astronomers such as Aristarchus of Samos, Hipparchus, and Claudius Ptolemy also used geometrical methods (*12*), while arithmetical methods are attested in the Antikythera mechanism (*14*) and in Greco-Roman astronomical papyri from Egypt (*15*). However, the Babylonian trapezoid procedures are geometrical in a different sense than the methods of the mentioned Greek astronomers, since the geometrical figures describe configurations not in physical space but in an abstract mathematical space defined by time and velocity (daily displacement).

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SUPPLEMENTARY MATERIALS

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