MA330
Sathaye

A few Kuṭṭaka Problems

Here are some problems which provide practice for the solutions of the linear integer equations of the form $ax - by = c$.

1 Class practice

1. Find a number (and all such positive numbers) which is divisible by 29 after adding 2 to it and also divisible by 45 after subtracting 7 from it.

   Do determine the smallest positive such number.

2. Find a number divisible by 3 after subtracting 5 and divisible by 31 after subtracting 7. (A small variation from Bhāskara I of 600 A.D.)

   Do determine the smallest positive such number.

3. Find a number which has remainder 5 when divided by 8, remainder 4 when divided by 9 and remainder 1 when divided by 7.

   Do determine the smallest positive such number.

   (A small variation from Bhāskara I of 600 A.D. Bhāskara I 600 A.D.)

4. Find a number which leaves a remainder 1 when divided by $2, 3, 4, 5$ or 6 but is divisible by 7.

   Do determine the smallest positive such number.

   (Āryabhaṭīya bhāṣya 600 A.D. , Ibn-al-Haitam 1000 A.D., Fibonacci 1202 A.D.)

5. Another variant is: Find a number which leaves a remainder 1 when divided by $2, 3, 4, 5, 6, 7, 8$ or 9 but is divisible by 11.

6. Suppose that if we count a set of objects by threes, then 2 are left; if we count by fives, then 3 are left and if we count by sevens, then 2 are left. How many objects are there? (It is implied that you should find the smallest such positive number. (Sunzi fifth century - the origin of the term "Chinese Remainder Theorem!")

7. The residue of the revolutions of Saturn is 24. Find the day number. (laghubhāskarīya 600 A.D.) Explanation: Let x be the day number or the number of days elapsed since the beginning of the yuga (actually the current Kali yuga).

   The planet Saturn makes $\frac{36,641}{394,479,375}$ revolutions in a day. . The statement that the "residue of revolutions is 24" means that the fractional number of revolutions is $\frac{24}{394,479,375}$. So, your task is to find (smallest) number of days elapsed and corresponding number of revolutions from the beginning of this yuga!

   By the way, this number of days in a yuga, which formally has $432,000$ years gives the average year as 365.259 solar days in length. The equation you get is

   $$36,641x - 24 = 394,479,375y$$

   where y is the number of complete revolutions.

8. The Sun's mean position was obtained as longitude 148 deg. 20 min. Calculate the day number. (Mahābhāskarīya 600 A.D.) and the number of revolutions of the Sun.

   For Sun, the reduced number of daily revolutions comes out to be $\frac{576}{210,389}$. The fractional number of revolutions is calculated to be $\frac{86,688}{210,389}$ after throwing away fractional part (less than 0.01) from the numerator. Verify this from the longitude.

2 Some problems with reduction tricks

1. Find a least positive integer n which is divisible by 63 and leaves a remainder of 90 when divided by 100. (Bhāskara II 1100+ AD).

2. Find a least positive integer n such that $221n + 65$ is divisible by 195. (Bhāskara II 1100+ AD).

3. Find a least positive integer n such that $100n \pm 90$ is divisible by 63. (Bhāskara II 1100+ AD).

4. Find a least positive integer n such that $5n - 7$ is divisible by 63 and $10n - 14$ is also divisible by 63.

   (Bhāskara II 1100+ AD).