

This is a take home exam. It should be completed and submitted **no later than March 30, 2012**. All questions should be worked out accompanied with explanations in words. Just numerical conclusions will be unacceptable.

There is no late submission and no “do-over” on this exam.

Separate questions should be worked on separate pages and stapled together.

1. (a) Develop a divisibility test by 37 patterned after Bhāratī Kṛṣṇa’s method. Explicitly **state what multiplier** you obtain.

(b) Be sure to illustrate and verify how the test works by testing a couple of small (6-7 digits) numbers, both divisible and not divisible by 37.

(c) Decide if 1351061075 is divisible by 37 using the test. Decide the divisibility of 1351064441105 by the same test.
2. Find the smallest positive integer x which
 - leaves a remainder of 2 when divided by 3 or 5 or 7,
 - leaves a remainder of 5 when divided by 11 or 13 and
 - is divisible by 17.

Also find the general solution for the same equations. It is important to show work. Answer by guessing is unacceptable!

3. Use the Brahmagupta-Jayadeva-Bhāskara method to find a smallest solution to the equation $x^2 - 167y^2 = 1$.

Be sure to show your work.

I need to see two answers: (1) The factored form as discussed. (2) The integer form after multiplying the factors and cleaning out the common factors.

Of course, calculators are expected to be used.

4. Use the Bruncker-Wallis algorithm to find a continued fraction expansion for $\sqrt{167}$. In this case, you may use the decimal approximation 12.92284798. It is enough to calculate only the first first 7 steps.