MA330 (Sathaye)
(161005) **Quiz 1** **Name:**______________________

The Brouncker-Wallis algorithm.

The aim of this exercise is to completely determine the (regular) continued fraction expansion of $\sqrt{19}$ using the algorithm discussed in the class.

This should be worked at home and submitted in class on Wednesday 10/12/16.

1. Start with the augmented matrix

$$M_0 = \left(\begin{array}{cc|cc} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & -19 \end{array}\right) \text{ so that the associated form is } f(x,y) = x^2 - 19y^2.$$

2. Carry out the algorithm steps to produce new matrices $M_1, M_2, \cdots$ etc. until the second part of the matrices repeats. **be sure to** explicitly record the value of t in a prominent place.

   Be sure to give complete details of how you are transforming one matrix to the next **in at least one step.** The remaining steps may be done without comment.

   Hint: There will be 7 steps and the 8th step will repeat the right hand side matrix of step 2.

Please turn over.

1. At the end of the above process, write down the obtained continued fraction expansion in the usual notation $[a_0; a_1; \cdots a_r]$. Further work is to be done on this continued fraction as asked below.

2. Determine **all the convergents.** See illustration at the bottom of this page.

   Using the calculator, get a decimal value for each convergent and compare these with the calculator value of the $\sqrt{19}$.

3. **Illustration:**

   Supposed that a continued fraction is $[2; 3; 2; 1; 3]$, then you first start with $\binom{0}{1}, \binom{1}{0}$. Then continue as usual to get:

$$\begin{array}{c|cccccc} \binom{0}{1} & \binom{1}{0} & \binom{2}{1} & \binom{7}{3} & \binom{16}{7} & \binom{23}{10} & \binom{85}{37} \\ & 2 & 3 & 2 & 1 & 3 & \end{array}$$

   Explanation: The pair $\binom{7}{3}$ is obtained by $3\binom{2}{1} + \binom{1}{0}$. The pair $\binom{16}{7}$ is $2\binom{7}{3} + \binom{2}{1}$. Continue. Make fractions from these to get the convergents (i.e. $2/1, 7/3, 16/7$ etc.)