

Use the Brahmagupta-Jayadeva-Bhāskara method to find a smallest solution to the equation

$$x^2 - 61y^2 = \pm 1.$$

Be sure to show your work. I need to see two answers:

- (1) The factored form as discussed.
- (2) The integer form after multiplying the factors and cleaning out the common factors.

(3) **Hint:** Do remember to use the two Brahmagupta tricks (when the extended norm reduces to ± 2 or ± 4 .) as discussed in class. You will need 2 or 7 steps depending on how well you use the tricks. ¹

¹This is the problem communicated by Fermat, but already solved by Bhaskara. The continued fraction for $\sqrt{61}$ comes out to be $[7, 1, 4, 3, 1, 2, 2, 1, 3, 4, 1, 14, 1, 4, 3, 1, 2, 2, 1, 3, 4]$. If you evaluate the usual convergents, then you have to use the first 22 steps!! The answer is huge: 1766319049, 226153980).