MA330 (Sathaye)
(140405) **Quiz 4** **Name:**_____

This is a take-home quiz. It should be submitted on or before Wednesday April 9.

1. Develop a divisibility test by 23 patterned after Bhāratī Kṛshṇa's method. Explicitly state what multiplier you obtain.

   Decide if 2452559 is divisible by 23 using the test.

   Decide the divisibility of a comparably large number of your choice by the same test.

2. Develop a divisibility test by 37 patterned after Bhāratī Kṛshṇa's method. Explicitly state what multiplier you obtain.

   Illustrate with a couple of fairly large numbers of your choice.

3. Work out the factorization technique of Nārāyaṇa to factor the number: $n = 259$.

   Illustrate the technique with two more three digit numbers of your choice.

   For your information, the technique is described here.

   Start with a number n which is odd. (Also a factor of 5 is easy to note and remove, so assume 5 does not divide n.) We illustrate with $n = 203$.

   - Start with a least choice of t such that $t^2 > n$. Set $s = t^2 - n$.
     Thus for $n = 203$, we start with $t = 15, s = 22$.
   - Check if s is a square. This can be done from a table of squares, or using a calculator.
     If $s = y^2$, then $n = t^2 - y^2$, so we have a factorization $n = (t - y)(t + y)$. Declare "Success"!!

     For our example, 22 is a non square, so continue.
   - If there is no success, then replace s by $s + 2t + 1$ and then replace t by $t + 1$. Check again.

     We change (t, s) to $(15 + 1, 22 + 2(15) + 1) = (16, 53)$. Since 53 is a non square, we repeat the process and get $(t, s) = (17, 86)$. Repeating the step, we get $(t, s) = (18, 121)$.

     Thus $203 = 18^2 - 11^2 = (7)(29)$.
   - When do we conclude that n must be a prime? This happens if $t - y = 1$ i.e. $t + y = n$ where $y^2 = s$.