MA330 (Sathaye)			
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## The Brouncker-Wallis algorithm. (due) 2/27/17

The aim of this exercise is to completely determine the (regular) continued fraction expansion of  $\sqrt{19}$  using the Brouncker-Wallis algorithm discussed in class.

1. Start with the augmented matrix

$$M_0 = \begin{pmatrix} 1 & 0 & | & 1 & 0 \\ 0 & 1 & | & 0 & -19 \end{pmatrix}$$
 so that the associated form is  $f(x, y) = x^2 - 19y^2$ .

2. Carry out the algorithm steps to produce new matrices  $M_1, M_2, \cdots$  etc. until the second part of the matrices repeats. **Be sure to** explicitly record the value of t at each step in a prominent place.

Be sure to record the appropriate row and column operations at each step.

**Hint Corrected:** There are only 7 steps needed before the right hand matrix repeats.

3. At the end of the above process, write down the obtained continued fraction expansion in the usual notation  $[a_0; a_1; a_2; \cdots]$ . Put a bar over the repeated part.

Further work is to be done on this continued fraction as asked.

Please turn over.

- 1. Copy the continued fraction expansion from the previous page.
- 2. Determine all the convergents for the *t*-list until repetition. See illustration at the bottom of this page.

Using the calculator, get a decimal value for each convergent and compare these with the calculator value of the  $\sqrt{19}$ . You should find further convergents at home to appreciate the approximation.

## 3. Illustration of work:

Supposed that a continued fraction is  $[2; \overline{3}; 2; 1]$ . Start with  $\binom{0}{1}, \binom{1}{0}, \binom{2}{1}$  as the second row and write the given *t*-values as the first row (starting in column 2).

$$\left(\begin{array}{ccccccccccc} 2 & 3 & 2 & 1 \\ \hline 0 & 1 & 2 & 7 & 16 & 23 \\ 1 & 0 & 1 & 3 & 7 & 10 \\ & & & & & \end{array}\right)$$

**Explanation: The pair**  $\binom{7}{3}$  is obtained by  $3\binom{2}{1} + \binom{1}{0}$ . The pair  $\binom{16}{7}$  is  $2\binom{7}{3} + \binom{2}{1}$ . Continue. Make fractions from these to get the convergents (i.e. 2/1, 7/3, 16/7 etc.)