The Man who Knew Infinity

Srinivasa Ramanujan Iyengar

(**Best known as S. Ramanujan**)

(22 Dec 1887 - 26 April 1920)

Life of Ramanujan

- This presentation is about life & achievements of famous mathematicians S. Ramanujan & is organized as below:

The Genius Mathematician

- S. Ramanujan hailed as an all-time great mathematician, like Euler, Gauss or Jacobi, for his natural genius, has left behind 4000 original theorems, despite his lack of formal education and a short life-span.
- Probably Ramanujan's life has no parallel in the history of human thought. He was a autodidact mathematical genius not only of the twentieth century but for all time to come.

Child Prodigy

- Learned college-level mathematics by age 11, and generated his own theorems in number theory and Bernoulli numbers by age 13 (including independently re-discovering Euler's identity).
- While in school, he was gifted George Schoobridge Carr’s 'Synopsis'. This book listed without proofs 4865 formulae in algebra, trigonometry, analytical geometry and calculus. Ramanujan not only proved for himself each of these formulae but also derived many new results and recorded them.

Child Prodigy

A thought of a 7 year old Prodigy

- Teacher: n / n =1, for every integer n.
- Ramanujan: “Is zero divided by zero is also one?”
- Ramanujan’s Answer: “Zero divided by zero may be anything. The zero of the numerator may be several times the zero of the denominator and vice versa”.

(7 year old prodigy was thinking of limits and limiting processes.)

Life Down & Ups

- After high school, Ramanujan passed a competitive examination in English and Mathematics and secured the Scholarship. He joined the Government Arts college, in the F.A. (First Examination in Arts, now a days BA I year) class at the age of 18 years.
- Due to his love & preoccupation with Mathematics, he could not pass in English and Sanskrit and hence was not promoted to the Senior F.A. Class.

Life Down & Ups

- He lost his Scholarship & forced to discontinue his studies due to this and the poverty of his family.
- However, he appeared privately for the F.A. examination but failed to pass the university examinations and this marked the end of his formal education.
- After having failed in his F.A. class at College, he ran from pillar to post in search of a benefactor for 6 (from 18 years of age to 24 years) years.

Life Down & Ups

- He tried for tutoring assignments to earn livelihood but failed. His love and dedication for math continued.
- He lived in absolute poverty. Once he said to one of his friends, “when food is problem, how can I find money for paper? I may require four reams of paper every month.”
- With the help of a recommendation letter from a mathematician, Ramanujan secured a clerical post in the 'Accounts' Section of the Madras Port Trust at an age of 24 years.

Life Down & Ups

- Meanwhile in 1911 at an age of 23 years, the first full length (15 page) research paper of Ramanujan, entitled: “Some properties of Bernoulli Numbers”, appeared in the Journal of the Indian Mathematical Society. This brought him to the forefront of international mathematics.
- After going through a paper by British mathematician Hardy, about some theorems on prime numbers, where he claimed that no definite expression has been found as yet, Ramanujan derived the expression which very nearly approximated to the real result.

Life Down & Ups

- Ramanujan communicated the derived results to Hardy through a letter, who after getting convinced that Ramanujan is a natural genius, persuaded university authorities to invite him to Cambridge.
- The orchestrated efforts of his admirers, resulted in inducing the Madras University, in offering him the first research scholarship of the University in May 1913; then in offering him a scholarship of 250 pounds a year for five years &100 pounds for passage by ship and for initial outfit to go to England in 1914 at an age of 26 years.

Life Down & Ups

- Ramanujan initially refused to go to England possibly due to his caste prejudice. But after mentoring from Neville, Ramanujan finally set sail for England at a age of 26 years in 1914.
- Prior to Ramanujan departure to England, Prof. Littlehailes & Mr. Arthur Davies arranged with the University for £ 60 (out of Scholarship amount of £ 250) per year to be sent to his parents in India. Thus, he fulfilled his responsibilities as the eldest son of the family.

Life Down & Ups

- At Cambridge, Ramanujan worked with Littlewood and Hardy, mathematicians of repute of that time.
- Ramanujan was awarded the B.A. degree by research (this degree was later renamed as PhD) in March 1916 at an age of 28 years, for his work on Highly Composite Numbers.
- He was elected a Fellow of the Royal Society of London in February 1918 at an age of 30 years. He was the second Indian to become a Fellow of the Royal Society (1st one was in 1841, around 80 years prior) & one of the youngest Fellows in the entire history of the Royal Society.

Life Down & Ups

- He was elected to a Trinity College Fellowship (a prize fellowship worth 250 pounds a year, which he was not destined to avail of.) He was the first Indian to be elected a Fellow of Trinity College, Cambridge.
- During his five year stay in Cambridge, Ramanujan published twenty one research papers containing theorems.

Prof. G. H. Hardy

- The role of G.H. Hardy (1877 - 1947), Professor of Mathematics at Trinity College, Cambridge, in the life and career of Ramanujan is immeasurable, peerless and beyond praise.
- Convinced that Ramanujan was a natural genius, Hardy made up his mind that Ramanujan should be brought to Cambridge.

Prof. G. H. Hardy

- The eccentric British mathematician G.H. Hardy is known for his achievements in number theory and mathematical analysis. But he is perhaps even better known for his adoption and mentoring of the self-taught mathematical genius, Ramanujan.
- Before writing to Hardy, Ramanujan had written to two well-known Cambridge mathematicians. But both of them had expressed their inability to help Ramanujan.

Prof. G. H. Hardy

- “Ramanujan was”, Hardy wrote, “my discovery. I did not invent him, like other great men, he invented himself, but I was the first really competent person who had the chance to see some of his work, and I can still remember with satisfaction that I could recognize at once what treasure I had found.”
- Some differences were also developed later between Ramanujan & Hardy due to working style.

Hardy-Ramanujan "taxicab numbers"

- A common anecdote about Ramanujan relates how Hardy arrived at Ramanujan's house in a cab numbered 1729, a number he claimed to be totally uninteresting. Ramanujan is said to have stated on the spot that, on the contrary, it was actually a very interesting number mathematically, being the smallest number representable in two different ways as a sum of two cubes. Such numbers are now sometimes referred to as "taxicab numbers".

Hardy-Ramanujan "taxicab numbers"

1,729 is the smallest number which can be represented in two different ways as the sum of two cubes:

$$\begin{aligned} 1729 &= 1^3 + 12^3 \\ &= 9^3 + 10^3 \end{aligned}$$

It is also incidentally the product of 3 prime numbers:

$$1729 = 7 \times 13 \times 19$$

The largest known similar number is:

$$\begin{aligned} 885623890831 &= 7511^3 + 7730^3 \\ &= 8759^3 + 5978^3 \\ &= 3943 \times 14737 \times 15241 \end{aligned}$$

The Lost Notebook

- Ramanujan recorded the bulk of his results in four notebooks of loose leaf paper. First 3 notebooks contained his work before leaving for England.
- Those 3 notebooks were published as a two-volume set in 1957 by the Tata Institute of Fundamental Research (TIFR), Mumbai, India.
- The fourth notebook with 87 unorganized pages, the so-called "lost notebook", was sent to Hardy by his wife after his death. The notebook remained untraced nearly for 40 years.

The Lost Notebook

- The “Lost Notebook” contained results he noted, during the last year of his life, after his return to India.
- This “Lost Notebook” was found by Prof. George E. Andrews of the Pennsylvania State University in 1976 in the trash.
- The “Lost Notebook” contained some 600 theorems, which Ramanujan called as “mock” theta functions.

The Lost Notebook

- The discovery of the “Lost” Notebook of Ramanujan contributed to a resurgence of interest in the life and work of Ramanujan.
- Its facsimile edition was brought out by Narosa Publishing House in 1987, on the occasion of Ramanujan's birth centenary nearly after 68 years of his death.

The Lost Notebook

- The results in his notebooks inspired numerous papers by later mathematicians trying to prove what he had found.
- He stated results that were both original and highly unconventional, and these have inspired a vast amount of further research.
- The many conjectures of Ramanujan opened up new areas of research in mathematics.

Misperception

- Ramanujan recorded the bulk of his results in four notebooks of loose leaf paper. These results were mostly written up without any derivations. Since paper was very expensive, Ramanujan would do most of his work and perhaps his proofs on **slate**, and then transfer just the results to paper. This is probably the origin of the misperception that Ramanujan was unable to prove his results and simply thought up the final result directly.

Misperception

- Professor Bruce C. Berndt of University of Illinois, who worked extensively on Ramanujan notebooks, during a lecture at IIT Madras in 2011 stated that, over the last 40 years, as nearly all of Ramanujan's theorems have been proven right.
- Also mathematicians agreed on the point that it was not possible for someone to imagine those results without solving / proving.

Personal Life

- Born to Kuppuswamy Srinivasa Iyengar & Komalathammal of Kumbakonam on Thursday, Dec 22, 1887, at 6 pm as their first child in Erode (Tamil Nadu, India).
- His nick name was Chinnaswamy, meaning Little Lord.
- At the age of five, Ramanujan had a bout of small pox, which left some permanent scars on his face.

Personal Life

- His mother use to sing at a local temple, from whom he learnt religious traditions, which remained part of his daily life till the end. He had a deep faith in God and remained vegetarian throughout his life.
- At the age of 21 in 1909, Ramanujan was married to Janaki, then a 10 year old girl.
- Ramanujan developed a rare disease, which could be treated with a surgical operation only. His family didn't have the money for the operation, but in1910, a doctor volunteered to do the surgery for free.

Personal Life

- Ramanujan suffered illnesses before and after his marriage and before his departure to England.
- His health was reasonably good during the first three years of his stay in Cambridge.
- The period of Ramanujan's stay in England almost overlapped with World War.
- Non availability of vegetarian food was one of factor behind his ill health.

Personal Life

- From May 1917, he had been in and out of hospital for duration of 2-3 weeks to 5-6 months until his departure to India in early 1919 (Last 2 years at Cambridge) due to suffering from Tuberculosis.
- Ramanujan was persuaded by Hardy to return to India with the hope that he would recover soon and return to Trinity College.
- After completing nearly five years at Cambridge, when Ramanujan appeared to have recovered sufficiently, he left England on 27th February 1919 for India at an age of 31.

Personal Life

- Unfortunately, his precarious health did not improve, on his return to India.
- Despite of the best medical attention from the doctors, his untimely end came on 26th April 1920, at Chetput, Madras, when Ramanujan was 32 years old.
- He was survived by wife Janaki, who lived until her death in 1994 in Chennai alongwith her foster son.

God to Him

- Ramanujan often quoted “An equation for me has no meaning unless it expresses a thought of God.”
- For example, 2^n-1 will denote the primordial God and several divinities. When n is zero the expression denotes zero, there is nothing; when n is 1, the expression denotes unity, the Infinite God. When n is 2, the expression denotes Trinity; when n is 3, the expression denotes 7, the Saptha Rishis and so on.

God to Him

- Ramanujan was eager to work out a theory of reality which would be based on the fundamental concepts of “zero”, “infinity” and the set of finite numbers.
- He spoke of “zero” as the symbol of the absolute (Nirguna-Brahmam) of the extreme monistic school of Hindu philosophy, that is, the reality to which no qualities can be attributed, which cannot be defined or described by words, and which is completely beyond the reach of the human mind.

God to Him

- According to Ramanujan, the appropriate symbol was the number “zero”, which is the absolute negation of all attributes.
- He looked on the number “infinity” as the totality of all possibilities, which was capable of becoming manifest in reality and which was inexhaustible.

God to Him

- According to Ramanujan, the product of infinity and zero would supply the whole set of finite numbers. Each act of creation, could be symbolized as a particular product of infinity and zero, and from each such product would emerge a particular individual of which the appropriate symbol was a particular finite number.

(As narrated by Prof. Prasantha Chandra Mahalanobius, the renowned Statistician, contemporary & good friend of Ramanujan at Cambridge)

Quotes about Ramanujan

- “Try to imagine the quality of Ramanujan's mind, one which drove him to work unceasingly while deathly ill, and one great enough to grow deeper while his body became weaker. I stand in awe of his accomplishments; understanding is beyond me. We would admire any mathematician whose life's work was half of what Ramanujan found in the last year of his life while he was dying".

Prof. Richard Askey

University of Wisconsin-Madison

Quotes about Ramanujan

- Hardy's personal ratings of mathematicians: Suppose that we rate mathematicians on the basis of pure talent on a scale from 0 to 100, Hardy gave himself a score of 25, Littlewood 30, Hilbert 80 and Ramanujan 100".

Bruce C Berndt

Distinguished Research Professor of Mathematics, University of Illinois

Best known for his work explicating the discoveries of Ramanujan. He is a coordinating editor of The Ramanujan Journal.

Quotes about Ramanujan

- Sheer intuitive brilliance coupled to long, hard hours on his slate made up for most of his educational lapse. This 'poor and solitary Hindu pitting his brains against the accumulated wisdom of Europe' as Hardy called him, had rediscovered a century of mathematics and made new discoveries that would captivate mathematicians for next century.

Robert Kanigel

Author of "The Man who Knew Infinity: A Life of the Genius Ramanujan"

Quotes about Ramanujan

- “I think it is fair to say that almost all the mathematicians who reached distinction during the three or four decades following Ramanujan were directly or indirectly inspired by his example.” Even those who do not know about Ramanujan’s work are bound to be fascinated by his life.

S. Chandrasekhar

The Indian born astrophysicist, who got Nobel Prize in 1983

Quotes about Ramanujan

- “I have to form myself, as I have never really formed before, and try to help you to form, some of the reasoned estimate of the most romantic figure in the recent history of mathematics, a man whose career seems full of paradoxes and contradictions, who defies all cannons by which we are accustomed to judge one another and about whom all of us will probably agree in one judgment only, that he was in some sense a very great mathematician.”

  Prof. Hardy

Quotes about Ramanujan

- “I found Hardy and Littlewood in a state of wild excitement because they believe, they have discovered a second Newton, a Hindu Clerk in Madras … He wrote to Hardy telling of some results he has got, which Hardy thinks quite wonderful.”

Bertrand Arthur William Russell

British philosopher& mathematician, Noble prize holder

Quotes about Ramanujan

- “Ramanujan’s conjectures formulated and their later generalization, have come to play a more central role in the mathematics of today, serving as a kind of focus for the attention of quite a large group of the best mathematicians of our time.
- The estimates of Ramanujan’s nature in mathematics certainly have been growing over the years. There is no doubt about that.

Prof. Atle Selberg

The Norwegian mathematician one of the great number theorists of this century

Quotes about Ramanujan

- S. Ramanujan, discovered by the Cambridge mathematician Hardy, whose great mathematical findings were beginning to be appreciated from 1915 to 1919. His achievements were to be fully understood much later, well after his untimely death in 1920. For example, his work on the highly composite numbers started a whole new line of investigations in the theory of such numbers."

Prof. Jayant Narlikar, Indian astrophysicist
In his book *Scientific Edge*

Quotes about Ramanujan

- “The fact that Ramanujan’s early years were spent in a scientifically sterile atmosphere, that his life in India was not without hardships that under circumstances that appeared to most Indians as nothing short of miraculous, he had gone to Cambridge, supported by eminent mathematicians, and had returned to India with very assurance that he would be considered, in time as one of the most original mathematicians of the century”.

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Eponym

Ramanujan is eponym of mathematical topics listed below:

- Ramanujan Magic Square
- Brocard-Ramanujan Diophantine equation
- Dougall-Ramanujan identity
- Hardy-Ramanujan number
- Landau-Ramanujan constant
- Ramanujan's congruences
- Ramanujan-Nagell equation
- Ramanujan-Peterssen conjecture
- Ramanujan-Skolem's theorem
- Ramanujan-Soldner constant
- Ramanujan summation
- Ramanujan theta function
- Ramanujan graph
- Ramanujan's tau function
- Ramanujan's ternary quadratic form
- Ramanujan prime
- Ramanujan's constant
- Ramanujan's sum
- Rogers-Ramanujan identity

Lessons Learnt / Unlearnt from Ramanujan's Life

- Fact, that despite the hardship faced by Ramanujan, he rose to such a scientific standing and reputation that no Indian has enjoyed before, should be enough for young Indians to comprehend that if they are deserving and can work hard, they can perhaps soar the way what Ramanujan had.

Lessons Learnt / Unlearnt from Ramanujan's Life

- Even today in India Ramanujan can not get a lectureship in a school / college because he had no degree. Many researchers / Universities will pursue studies / researches on his work but he will have to struggle to get even a teaching job.

Lessons Learnt / Unlearnt from Ramanujan's Life

- Even after more than 90 years of the death of Ramanujan the situation is not very different as far the rigidity of the education system is concerned. Today also a 'Ramanujan' has to clear all traditional subjects' exams to get a degree irrespective of being genius in one or more different subjects.

Lessons Learnt / Unlearnt from Ramanujan's Life

- He was offered a chair in India after becoming a Fellow of the Royal Society. But it is disgraceful that India's talent has to wait for foreign recognition to get acceptance in India or else immigrate to other places. Similarly many other scientists had no other option but to immigrate for opportunities & recognition. Many of those won international recognitions including noble prizes. The process of this brain drain is still continuing even after 60 years of independence.

Lessons Learnt / Unlearnt from Ramanujan's Life

- The most important lesson that we should draw from Ramanujan's life about the educational system is that systems / provision should be made to support specially gifted children with very strong interests in one direction, at all stages of the educational system.

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Ramanujan Song

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