

The Brouncker-Wallis algorithm.

The aim of this exercise is to completely determine the (regular) continued fraction expansion of $\sqrt{19}$ using the algorithm discussed in the class.

This should be worked at home and submitted in class on Monday 3/10/14.

1. Start with the augmented matrix

$$M_0 = \left(\begin{array}{cc|cc} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & -19 \end{array} \right) \text{ so that the associated form is } f(x, y) = x^2 - 19y^2.$$

¹

2. Carry out the algorithm steps to produce new matrices $M_1, M_2, \dots$ etc. until the second part of the matrices repeats. **be sure to** explicitly record the value of t in a prominent place.

Be sure to give complete details of how you are transforming one matrix to the next **in at least one step**. The remaining steps may be done without comment.

Hint: There will be 7 steps and the 8th step will repeat the right hand side matrix of step 2.

Please turn over.

¹In the earlier posting, this was incorrectly typed as $x^2 - 11$. Corrected on 3/9/14 thanks to Brittany's inquiry.

1. At the end of the above process, write down the obtained continued fraction expansion in the usual notation $[a_0; a_1; \cdots a_r]$. Further work is to be done on this continued fraction as asked below.
2. Determine **all the convergents**. See illustration at the bottom of this page.

Using the calculator, get a decimal value for each convergent and compare these with the calculator value of the $\sqrt{19}$.

3. Illustration:

Supposed that a continued fraction is $[2; 3; 2; 1; 3]$, then you first start with $\binom{1}{0}, \binom{2}{1}$. Then continue as usual to get:

$$\binom{1}{0} \quad \binom{2}{1} \quad \binom{7}{3} \quad \binom{16}{7} \quad \binom{23}{10} \quad \binom{85}{37}$$

$$2 \quad 3 \quad 2 \quad 1 \quad 3$$

Explanation: The pair $\binom{7}{3}$ is obtained by $3\binom{2}{1} + \binom{1}{0}$. The pair $\binom{16}{7}$ is $2\binom{7}{3} + \binom{2}{1}$. **Continue.** Make fractions from these to get the convergents (i.e. $2/1, 7/3, 16/7$ etc.)