

Review for Exam 2 - Part I

1 Inequalities

1.1 Equivalent Inequalities

1.1.1 Example

For each pair of inequalities, determine if the two inequalities are equivalent. Explain your reasoning.

(a) $4 - x < 1$ and $x > 3$.

Yes, $4 - x < 1 \rightarrow 4 - 1 < x \rightarrow 3 < x$ or $x > 3$.

(b) $\frac{x}{x+3} > 7$ and $x > 7(x+3)$.

No We can't multiply by expressions since the sign can change.

1.2 Solving a Linear and Nonlinear Inequalities.

1.2.1 Example

Solve the inequality $4x + 2 \geq 2 + 5x$.

$$4x + 2 \geq 2 + 5x$$

Combine everything with x on one side and the rest on the other. Such as

$$4x - 5x \geq 2 - 2$$

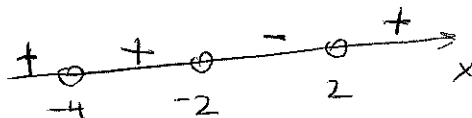
$$-x \geq 0$$

$$\boxed{x \leq 0}$$

1.2.2 Example

Solve the inequality $(x+2)(x-2)^3(x+4)^2 > 0$. Write your answer in interval notation.

First, consider equality $(x+2)(x-2)^3(x+4)^2 = 0$. Then by zero product property $x = -2$, $x = 2$, and $x = -4$. Then plot them on the number line:



Solution: $(-\infty, -4) \cup (-4, -2) \cup (2, +\infty)$

1.2.3 Example

Solve the inequality $x^2 + x > 12$. Write your answer in interval notation.

$$x^2 + x > 12 \rightarrow x^2 + x - 12 > 0$$

Consider equality $x^2 + x - 12 = 0$ which factors into

$$(x+4)(x-3) = 0 \text{ and using zero product property } x = -4 \text{ and } x = 3$$

Put them on the number line and determine signs:



Solution: $(-\infty, -4) \cup (3, +\infty)$

1.2.4 Example

Solve the inequality $x^2 + x > 12$. Write your answer in interval notation.

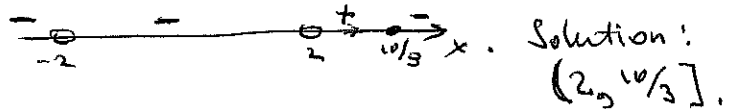
1.2.5 Example

Solve the inequality $\frac{1}{x-2} \geq \frac{4}{x+2}$. Write your answer in interval notation.

$$\frac{1}{x-2} \geq \frac{4}{x+2} \longrightarrow \frac{1}{x-2} - \frac{4}{x+2} \geq 0 \longrightarrow \frac{x+2-4(x-2)}{(x-2)(x+2)} \geq 0$$

$$\longrightarrow \frac{x+2-4x+8}{(x-2)(x+2)} \geq 0 \longrightarrow \frac{10-3x}{(x-2)(x+2)} \geq 0. \text{ Note: } x \neq \pm 2 \text{ cause us division}$$

by zero and if we look at the top $10-3x \geq 0 \longrightarrow \frac{10}{3} \geq x$ (so $x = \frac{10}{3}$ is giving us zero). So the number line



1.2.6 Example (Challenging)

Solve the inequality $|x-2|+3 < 6$. Write your answer in interval notation.

$$|x-2|+3 < 6 \longrightarrow |x-2| < 3$$

$$\longrightarrow -3 < x-2 < 3$$

or

$$\longrightarrow -3 < x-2 \text{ and } x-2 < 3$$

$$\longrightarrow -3+2 < x < 3+2$$

$$\longrightarrow -1 < x < 5$$

Solution: $(-1, 5)$.

