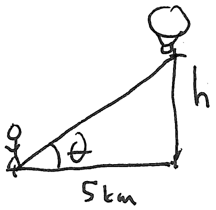
MXC Worksheet 6 Solutions

1) a) T b) F c) T d) T e) F f) F g) T

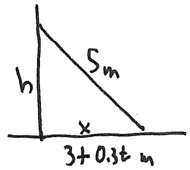
2)



a) $h = 5\tan\theta$ km

b) $\frac{dh}{dt} = 5\sec^2\theta \cdot \frac{d\theta}{dt} = 5\sec^2\left(\frac{\pi}{3}\right)\cdot 0.1$ km/min
$= 2$ km/min.

3)



(t in s)

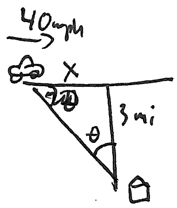
$h_0^2 + 3^2 = 5^2$

$\boxed{h_0 = 4\text{ m}}$

$\frac{dh}{dt} = -\frac{x}{h}\frac{dx}{dt} = -\frac{3}{4}(0.3) = \boxed{-\frac{9}{40}\text{ m/s}}$

-0.225 m/s

4)



$\tan\theta = \frac{x}{3}$

$3\sec^2\theta\,\frac{d\theta}{dt} = \frac{dx}{dt}$

$\frac{d\theta}{dt} = \frac{1}{3}\cos^2\theta\,\frac{dx}{dt} = \frac{1}{3}\cdot\cos^2(0)\cdot 40 = \boxed{\frac{40}{3}\text{ rad/hr}}$

5) $f(x) = \frac{1}{(x-3)^2}$

$$f'(x) = \lim_{h\to 0}\frac{\frac{1}{(x+h-3)^2} - \frac{1}{(x-3)^2}}{h} = \lim_{h\to 0}\frac{(x-3)^2 - (x+h-3)^2}{h(x-3)^2(x+h-3)^2} = \lim_{h\to 0}\frac{(x^2-6x+9) - (x^2+2hx-6x+h^2-6h+9)}{h(x-3)^2(x+h-3)^2}$$

$$= \lim_{h\to 0}\frac{-2hx-h^2+6h}{h(x-3)^2(x+h-3)^2} = \lim_{h\to 0}\frac{-2x+6-h}{(x-3)^2(x+h-3)^2} = \frac{-2(x-3)}{(x-3)^2(x-3)^2} = \boxed{-\frac{2}{(x-3)^3}.}$$

6) $f(x) = e^{x^2}$, $a = 1$.

7) $f(\theta) = \cos\theta - \sin\theta$, $a = \pi$.

8) $g'(7) = \frac{1}{f'(g(7))} = \frac{1}{f'(3)} = \boxed{\frac{1}{\sqrt{2}}.}$

9) $\sin(y^2e^y) = x^2 + y - 1$

$\cos(y^2e^y)(2y\,e^y + y^2e^y)y' = 2x + y'$

$y' = \frac{2x}{\cos(y^2e^y)(2y+y^2)e^y - 1}$

At $(-1,0)$:

$y' = \frac{-2}{\cos(0)\cdot 0\cdot 1 - 1} = 2.$

$y - 0 = 2(x+1).$

$\boxed{y = 2x + 2}$

10 | tangent line

$y = 3x - 14$

so $f'(4) = 3$

$f(4) = y(4) = -2$

11 | a) $f(x) = (2\pi)^{x^2} + \frac{1}{\sqrt{x}}$

$f'(x) = (2\pi)^{x^2} \cdot (\ln 2\pi) \cdot 2x + -\frac{1}{2}x^{-3/2}$

b) $8^{2x - \frac{3}{x}} \cdot \ln 8 \cdot (2 + \frac{3}{x^2}) = f'(x)$

c) $g(x) = \ln\left(\frac{x^2 - 2x + 1}{x}\right) = 2\ln(x-1) - \ln(x)$

$g'(x) = \frac{2}{x-1} - \frac{1}{x}$

12 | $f(x) = 3\cos(x) - \sin(2x)$

$f^{(7)}(x) = 3\sin(x) + 2^7\cos(2x)$.

13 | a) $l(x) = \sec(7x)$

$l'(x) = 7\sec(7x)\tan(7x)$

$l''(x) = \cancel{7}\,49[\sec(7x)\tan^2(7x) + \sec^3(7x)]$

b) $m(x) = \cot(x^2)$

$m'(x) = -2x\csc^2(x^2)$

$m''(x) = -2\csc^2(x^2)$ $\cancel{+8x\,cs}$

$-2x(2\csc(x^2)(-\csc(x^2)\cot(x^2)) \cdot 2x)$

$= 2\csc^2(x^2)[4x^2\cot(x^2) - 1]$

14 | $\frac{dV}{dt} = \frac{3}{2}\pi$ cm³/s, spherical balloon

a) $V = \frac{4}{3}\pi r^3$

$\frac{dV}{dt} = \underset{cm^2}{4\pi r^2} \cdot \frac{dr}{dt}$ cm/s

$\frac{3}{2}\pi = 4\pi(7)^2 \cdot \frac{dr}{dt}$

$\frac{dr}{dt} = \frac{3}{392}$ cm/s

≈ 0.007653 cm/s

b) increasing

volume ↑, so r ↑

c) $SA = 4\pi r^2$

$\frac{d(SA)}{dt} = 8\pi r \frac{dr}{dt}$

$= 8\pi(7) \cdot \frac{3}{392} = \frac{3\pi}{7\cancel{49}} = \boxed{\frac{3\pi}{343} \text{ cm}^2/s}$

≈ 0.027477 cm²/s

d) Increasing.

e) $\frac{dV}{dt} = \frac{-\pi}{2}$ cm³/s. $V = 36\pi = \frac{4}{3}\pi r^3$

$3^3 = r^3$

$r = 3$ cm

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$-\frac{\pi}{2} = 4\pi \cdot 9 \cdot \frac{dr}{dt}$

$\boxed{\frac{dr}{dt} = -\frac{1}{72} \text{ cm/s}}$

• − sign means radius is shrinking

15 $h(t)=at^2+bt+c$

$v(0)=0\ m/s$

$h(0)=64\ m$

$a(t)=-8\ m/s^2$

a) $h(0)=c=64$ $\boxed{h(t)=-4t^2+64\ m}$

$v(0)=2a(0)+b=0$

$b=0$

$a(t)=2a=-8$

$a=-4$

b) $0=h(t)=-4t^2+64$

$t^2=16$

$\boxed{t=4\ s}$

c) $v(4)=-8(4)=\boxed{-32\ m/s}$

16 $3x^2+4y^2+3xy=24$

$6x+8yy'+3y+3xy'=0$

$(8y+3x)y'=-6x-3y$

$y'=\dfrac{-6x-3y}{8y+3x}$

$y'=0$ if $-6x-3y=0$ and $8y+3x\neq 0.$

$\Downarrow$

$y=-2x$ and $y\neq -\frac{3}{8}x$ (so $(x,y)\neq(0,0)$, but this is ok b/c $(0,0)$ is not on the graph)

Now if $y=-2x$ we have

$3x^2+4(-2x)^2+3x(-2x)=24$

$(3+16-6)x^2=24$

$x^2=\frac{24}{13}$

$x=\pm\sqrt{\frac{24}{13}}$

So $\boxed{\left(\sqrt{\tfrac{24}{13}},-2\sqrt{\tfrac{24}{13}}\right) \text{ and } \left(-\sqrt{\tfrac{24}{13}},2\sqrt{\tfrac{24}{13}}\right)}$

17] $f(x) = 17x^2 + 6x + xe^{2x} - 9$.

Find all x so that $f^{(3)}(x) = 0$.

$$f'(x) = 34x + 6 + (1+2x)e^{2x}$$

$$f''(x) = 34 + (2 + 2(1+2x))e^{2x}$$

$$= 34 + (4 + 4x)e^{2x}$$

$$f^{(3)}(x) = (4 + 2(4+4x))e^{2x}$$

$$= (12 + 8x)e^{2x}$$

$$f^{(3)}(x) = 0 = (12 + 8x)e^{2x}$$

$$12 + 8x = 0$$

$$x = -\frac{12}{8}$$

$$\boxed{x = -3/2}$$