

Quiz #5

Directions: Carefully read each question below and answer to the best of your ability in the space provided. You **MUST** show your work to receive full credit!

1. (5 points) Find the limit

$$\lim_{x \rightarrow \infty} \frac{\cos^2(x)}{x^5}$$

using the sandwich theorem.

Solution: We know that $\cos(x)$ is bounded between -1 and 1 (i.e. $-1 \leq \cos(x) \leq 1$ for any $x \in \mathbb{R}$) but when you square it, we get that

$$0 \leq \cos^2(x) \leq 1,$$

since any number squared is not negative number. Thus we get

$$0 \leq \cos^2(x) \leq 1 \quad \rightsquigarrow \quad \frac{0}{x^5} \leq \frac{\cos^2(x)}{x^5} \leq \frac{1}{x^5} \text{ and}$$

$$0 = \lim_{x \rightarrow \infty} 0 = \lim_{x \rightarrow \infty} \frac{1}{x^5} = 0.$$

Thus, by the sandwich theorem, we can conclude that

$$\lim_{x \rightarrow \infty} \frac{\cos^2(x)}{x^5} = 0.$$

2. (5 points) Let

$$f(x) = 12x^3 + 11x^2 + 34x - 65.$$

Show using the intermediate value theorem, that f has a zero in the interval $[0, 2]$. Then use the intermediate value theorem, determine if this zero lies in $[0, 1]$ or $[1, 2]$.

Solution: Let's verify first that function f has a zero in the interval $[0, 2]$. That's

$$f(0) = 12(0)^3 + 11(0)^2 + 34(0) - 65 = -65 \quad \text{and}$$

$$f(2) = 12(2)^3 + 11(2)^2 + 34(2) - 65 = 143.$$

Zero is in the interval $[-65, 143]$ and since f is continuous, by the intermediate value theorem we know that there exists $c_1 \in [0, 2]$ such that $f(c_1) = 0$, so we have a zero in the interval $[0, 2]$.

Now, if we bisect interval into intervals $[0, 1]$ or $[1, 2]$, then

$$f(1) = 12(1)^3 + 11(1)^2 + 34(1) - 65 = -8.$$

So $f(0) = -65$ and $f(1) = -8$, zero is not in the interval $[-65, -8]$, therefore, the intermediate value theorem is not applicable to conclude that our function f has a zero in $[0, 1]$ but since $f(1) = -8$ and $f(2) = 143$, then zero is in the interval $[-8, 143]$ and since f is continuous, by the intermediate value theorem we know that there exists $c_2 \in [1, 2]$ such that $f(c_2) = 0$, so we have a zero in the interval $[1, 2]$.

Name: _____

Section (circle one): 003 004

Question:	1	2	Total
Points:	5	5	10
Score:			