1. Let $E(1)_* = A_*/(z_1^2, z_2^2, z_3, z_4, \ldots) = E(z_1, z_2)$. This is a quotient Hopf algebra of $A_*$. Let $E(1)$ be the dual of $E(1)_*$, with $Q_0 = z_1^\vee$ and $Q_1 = z_2^\vee$. We have Hopf maps

$$A_* \twoheadrightarrow E(1)_*, \quad E(1) \hookrightarrow A.$$ 

Express the image of $E(1)$ in the admissible basis, and show that $E(1) \cong E(Q_0, Q_1)$ is an exterior (commutative) algebra. Furthermore, show that $Q_0$ and $Q_1$ are primitive.

2. Let $A(1)_* = A_*/(z_1^4, z_2^2, z_3, z_4, \ldots) = \mathbb{F}_2[z_1, z_2]/(z_1^4, z_2^3)$. This is a quotient Hopf algebra of $A_*$. Let $A(1)$ be the dual of $A(1)_*$. We have Hopf maps

$$A_* \twoheadrightarrow A(1)_*, \quad A(1) \hookrightarrow A.$$ 

Show that $z_n^\vee = [n]$ for $n = 1, 2, 3$, and more generally express the image of $A(1)$ in the admissible basis.

3. There is an operation referred to as “stripping”, which allows you to get away with remembering only the single family of Adem relations $2n - 1 \equiv 0$. The idea is to consider the composition

$$A_* \otimes A \xrightarrow{id \otimes \Delta} A_* \otimes A \otimes A \xrightarrow{\text{eval} \otimes id} \mathbb{F}_2 \otimes A.$$ 

The stripping procedure says that since $2n - 1 \equiv 0$, it follows that this composition sends $z_1 \otimes 2n - 1 \equiv 0$. So evaluating this image gives a new relation in $A$.

Thus $[3][2] = 0$ implies that $[2][2] + [3][1] = 0$. The stripping procedure says that from the trivial product $[2n - 1][n]$, you can “strip” one from each square in turn, and retain a valid relation. Use stripping to compute $[8][6]$. 